Midterm Answer Key
110 B Spring '99

1) A) We know in a Malthusian Equilibrium (population with a stable economy):

$$B = D = \frac{1}{e_0}$$

$$\frac{25}{1000} = \frac{1}{e_0} \Rightarrow e_0 = 40$$

B) Life expectancy depends only on the birth rate, the Birth rate depend only on fertility, since fertility did not change, the birth rate did not change, and hence life expectancy did not change.

C) Life expectancy depends only on the birth rate, the Birth rate depend only on fertility, since fertility did not change, the birth rate did not change, and hence life expectancy did not change. (Yes, it is the exact same answer as above.)

D) Population Growth = B – D D=20/1000=2%, Pop Growth =3%
B = Pop. Growth + D = 5%=50/1000

2) A) We know that the output per worker in agriculture at least doubled during the industrial revolution because of three facts: 1) Population more than doubled, hence the number of people who needed to be fed doubled; 2) Wages/Incomes increased, so by Engels law, we know food consumption per person increased; and 3) The share of the population in Agriculture decreased.

B) We know that output per acre in agriculture at least doubled during the industrial revolution because of three facts: 1) Population more than doubled, hence the number of people who needed to be fed doubled; 2) Wages/Incomes increased, so by Engels law, we know food consumption per person increased; and 3) the amount of arable land in England did not increase.

3) a)

Year 0 $PQ = rk + wl + st$

Year 1 $(P + \Delta P)(Y + \Delta Q) = (r + \Delta r)(k + \Delta k) + (w + \Delta w)(l + \Delta l) + (s + \Delta s)(t + \Delta t)$

F.O.I.L. Year 1

$PQ + P\Delta Q + \Delta PQ + \Delta P\Delta Q = rk + r\Delta k + \Delta rk + \Delta r\Delta k + wl + w\Delta l + \Delta wl + \Delta w\Delta l + st + s\Delta t + \Delta st + \Delta s\Delta t$

We assume any term which has (2) Δs is equal to 0, also PQ on the LHS cancels with rk + wl + st on the RHS

We next Divide both sides by PQ :

$$\frac{\Delta \mathrm{P}}{\mathrm{P}} + \frac{\Delta \mathrm{Q}}{\mathrm{Q}} = \frac{\Delta \mathrm{rk}}{\mathrm{PQ}} + \frac{r\Delta \mathrm{k}}{\mathrm{PQ}} + \frac{\Delta \mathrm{wl}}{\mathrm{PQ}} + \frac{w\Delta \mathrm{l}}{\mathrm{PQ}} + \frac{\Delta st}{\mathrm{PQ}} + \frac{s\Delta t}{\mathrm{PQ}}$$

On the RHS : each term has a Δ in it (say Δx). multiply that term by $\frac{\mathrm{x}}{\mathrm{x}}$:

$$\frac{\Delta \mathrm{P}}{\mathrm{P}} + \frac{\Delta \mathrm{Q}}{\mathrm{Q}} = \frac{\Delta \mathrm{rk}}{\mathrm{PQ}}\left(\frac{r}{r}\right) + \frac{r\Delta \mathrm{k}}{\mathrm{PQ}}\left(\frac{k}{k}\right) + \frac{\Delta \mathrm{wl}}{\mathrm{PQ}}\left(\frac{w}{w}\right) + \frac{w\Delta \mathrm{l}}{\mathrm{PQ}}\left(\frac{l}{l}\right) + \frac{\Delta st}{\mathrm{PQ}}\left(\frac{s}{s}\right) + \frac{s\Delta t}{\mathrm{PQ}}\left(\frac{t}{t}\right)$$

rearragne

$$\frac{\Delta \mathrm{P}}{\mathrm{P}} + \frac{\Delta \mathrm{Q}}{\mathrm{Q}} = \frac{\mathrm{rk}}{\mathrm{PQ}}\left(\frac{\Delta r}{r}\right) + \frac{r\mathrm{k}}{\mathrm{PQ}}\left(\frac{\Delta k}{k}\right) + \frac{\mathrm{wl}}{\mathrm{PQ}}\left(\frac{\Delta w}{w}\right) + \frac{w\mathrm{l}}{\mathrm{PQ}}\left(\frac{\Delta l}{l}\right) + \frac{st}{\mathrm{PQ}}\left(\frac{\Delta s}{s}\right) + \frac{st}{\mathrm{PQ}}\left(\frac{\Delta t}{t}\right)$$

$\mathrm{g_P} + \mathrm{g_Q} = \alpha \mathrm{g_r} + \alpha \mathrm{g_k} + \beta \mathrm{g_w} + \beta \mathrm{g_l} + \gamma \mathrm{g_s} + \gamma \mathrm{g_t}$

rearrange

$g_Q - \alpha g_k - \beta g_l - \gamma g_t = \quad \alpha g_r + \beta g_w + \gamma g_s - g_p$

We know

$\mathrm{g_A} = g_Q - \alpha g_k - \beta g_l - \gamma g_t$

so

$\mathrm{g_A} = \alpha g_r + \beta g_w + \gamma g_s - g_p$

Q.E.D

b)

$$g_A = \alpha g_r + \beta g_w + \gamma g_s - g_p$$

$$= .4(1\%) + .5(2\%) + .1(3\%) - (-3\%) = 4.7\%$$

c) Yes, the same error exists. This can be explained two ways. First, in the above proof, we see that the prices equation and the quantities equation are equal, hence they supply the same estimate of g_A. Since, we know that the quantities equations overestimates g_A. it must also be true that the prices equation does as well. Second, the reason that g_A is overestimated in the quantities equation is because $\propto$ is underestimated. The same $\propto$ is used in the price equation, hence the same error occurs.

4) Note the answer to this long question is not in essay form, and instead of completely answering the question, it will put you on the right track.

   This question has two parts: 1) similarities of inventions; and 2) examples (i.e. list inventions, inventors, and possibly dates). Ideally part 2 augments your answer to part 1. However, if people just listed or discussed most of the major inventions during the Industrial revolution, especially in the cotton textile industry, you would have done well on the example part. The "examples" part is not listed here, you should look in your notes to find these.

   You needed to list at least 4 of the following similarities: (Many of these similarities are very closely related to each other)

   1) Inventors were mainly tinkerers w/o formal training.
   2) Inventors received little or no profit with the exception of one inventor.
   3) Inventors were not able to utilize the patent system well, because of flaws in the English patent system.
   4) Inventions were simple machines, once another person saw the invention, it could easily be duplicated
   5) Most inventions could have been invented at any time earlier in history, i.e. the Romans could have done it: (no new tech came about which these invention used)
   6) The inventions were not very related to each other i.e. The Spinning Jenny did not need the loom, nor did the loom need the spinning jenny,
   7) Most of these inventions occurred in the North of the U.K. (North England and Scotland)

Similarities that many of you put that may seem correct, and the reasons why they received no credit:

a) the inventions mainly occurred between 1770 and 1850. This is the equivalent as saying that most of the inventions occurred during the Industrial Revolution, however that was part of the question.
b) These inventions increased productivity and were labor saving: Yes, again this is true, however, almost all inventions (at least the ones that anyone would study) increase productivity, so while this is true, it is not a similarity of note for this time period that is not of note for all other time periods.