

**PROBLEM SET #6: PRODUCTION COSTS, PERFECT COMPETITION,
MONOPOLY, PRICE DISCRIMINATION**

Notes: If the total cost function of a firm has the form $TC = a + bq + cq^2$, then the marginal cost of the firm is $MC = b + 2cq$.

Competitive Markets

1. Suppose Bella's Birkenstocks produces sandals in the perfectly competitive sandal market. The total cost of production in the short run is $STC = 64 + q^2$. The long run total cost LTC is also $64 + q^2$, except that $LTC = 0$ at $q = 0$ in the long run (i.e. $LTC(0) = 0$, $LTC(1) = 65$, $LTC(2) = 69$ etc.).

a. What are SATC, SAVC and SMC? **SATC = $64/q + q$, SAVC = q , SMC = $2q$**

b. If the price of sandals is \$32, what is Bella's production? **q=16.**
What is the economic profit? **\$192**

c. If the price for sandals were \$8, what is Bella's production? **q= 4**
What is her economic profit? **-\$48**
Should she shut down?

No, because price is still greater than SAVC at q=4 which is \$4. Looking at it another way, if she did shut down, her profit would be even lower at -\$64.

d. Is there any price which would cause Bella to shut down in the short run?

No. For any price p, $SMC=p$ implies $2q=p$ or $p = 2 \times SAVC$. Thus, any positive price will always be larger than SAVC and therefore Bella should only shut down with $p=0$.

e. What is Bella's short run supply curve?

From the lack of a shut-down point and $MC=p$, we have $S(p) = 2q$.

3. In the short run there are 19 other sandal producers, each with the same costs as Bella.

a. What is industry output at a price of \$32? $Q = 16 \times 20 = 320$

b. What is the industry short run supply curve?

$$q=(0.5)p, Q = 20q = 20(0.5)p = 10p, S(p) = 10p$$

c. If the demand for sandals is $Q = 640 - 10P$, how many sandals are sold in the short run with 20 producers? $D(p) = S(p): 640 - 10p = 10p, p = \$32 \text{ \& } Q = 320$

What is the profit earned by each company?

\$192 (p=\$32 implies same conditions as problem 2b.)

d. If the sandal industry is a **constant cost** industry in the long run, what is the long run price and quantity? $LMC=LATC: 2q = 64/q + q, q=8, p = MC = 2q = 16.$

How many firms are there in the industry? $Q = D(16) = 640 - 160 = 480,$
 $\# \text{ firms} = Q/q = 480/8 = 60$

e. Is the constant cost assumption reasonable? **Yes, not specialized labor**

4. Which of the following industries will be **constant cost** competitive industries, and which **increasing cost**.

a. Wheat production? **Increasing Cost: Because most land is not particularly good for wheat production, the marginal productivity of each additional acre will fall while the cost will rise for compensating inputs such as irrigation. (Land is a specialized input.)**

b. Wine production? **Increasing Cost (same as above).**

c. Paper clip production? **Constant Cost: paper clip production requires a very small part of steel output and can therefore expect a constant price for steel, no matter how many paper clips are produced. (The steel in paper clips is not a specialized input.)**

d. Paper cup production? **Constant Cost:: similar reasoning except we are now concerned with paper-pulp and the trees used to make paper pulp instead of steel.**

5. What is **producer surplus** in a constant cost competitive industry in the long run? Explain.

Zero: Long run supply curve is a horizontal line equal to minimum long run average cost (such as \$16 above with problem 3d) Therefore, since there is no area between price and the supply curve, there is no producer surplus.

6. The US shirt industry is perfectly competitive and is in long-run equilibrium. There are 10 firms each with a total cost function of $STC = 9 + q^2$. The long run total costs are the same except that the fixed costs are not incurred if the firm does not produce. The number of firms is fixed in the short run, but can change in the long run. Imports are supplied with infinite elasticity at $P = \$8$.

- a. Draw the long run average and marginal cost curve of one US firm. At what quantity is LAC minimized?

See separate page for graph. LAC is minimized when $LAC=MC: 9/q + q = 2q, q=3$.

- b. Assuming the industry is constant cost in the long run draw the domestic industry long-run supply curve. On the same graph draw the domestic industry short-run supply curve.

See separate page for graph. Long run domestic supply curve is found by setting price equal to marginal cost at minimum LAC: $p=(2)(3)=6$, this supply curve is a horizontal line at $p=6$. In the short run, the supply curve of each firm is $S(p) = (0.5)q$ (just like problem 2e). For the industry, $Q = 10q = 10(0.5)p = 5p$, so that $S(p) = 5p$ (very much like problem 3b).

- c. Draw the short-run and long-run total supply curve (including imports).

See separate page for graph. In the long run, the domestic industry will supply any quantity for a price of \$6, and therefore no one will buy the \$8 imports, leaving the long-run supply curve as a horizontal line at \$6. Put another way, at a price of \$6, importers will supply zero shirts, so that the long run total supply curve will be identical to the long run domestic supply curve.

In the short run, as long as the domestic supply curve is below \$8, importers will not sell any shirts and the total supply curve will be identical to the domestic supply curve. However, if the domestic supply curve is at or above \$8, then the importers will supply shirts at \$8 each, and the total supply curve will remain flat at \$8 (see graph). The cutoff point for these two parts of the curve is $Q = S(8) = (5)(8) = 40$.

- d. Suppose the demand curve is $Q_d = 150 - 10P$. What is price, quantity supplied domestically, and imports in the short-run? In the long-run?

Short run. With only domestic supply, $D(p)=S(p): 150 - 10p = 5p$, $p=\$10$. Since this is greater than the \$8 price at which importers are willing to supply, the price will be \$8 and total demand will be $D(8) = 70$ shirts. Domestic producers will then supply 40 shirts (calculated above in part (e)) and importers will supply the remaining 30 shirts.

Long run. In the long run, domestic supply is defined $p = \$6$, so that domestic quantity is given by $D(6) = 90$ shirts. We have already seen that importers will not supply any quantity at this price (so that a total of 90 shirts are supplied in the long run).

- e. Describe the adjustment process from the short-run to the long-run.

In the short run, each firm will supply $(0.5)(8) = 4$ shirts, and make an (economic) profit of $(8)(4) - (9+16) = \$7$. Over time, this excess profit will attract other firms to the industry, shifting the domestic supply curve to the right. Starting with the 18th firm, domestic producers will drive price below \$8 with ever lower profits. As price approaches \$6, the profit of individual firms will approach zero.

- f. Ross Perot proposes banning shirt imports. Who would gain and who lose in the short-run? What would be the short-run deadweight loss?

Without import competition, market equilibrium will be determined by the demand and short run domestic supply curves, with $p=\$10$ and $Q=(5)(10) = 50$ shirts (see part (d)). Then each firm will supply 5 shirts with a profit of $(10)(5) - (9+25) = \$16$. Clearly the firms win with these larger profits, and consumers lose because of the higher price. Consumers also lose because for many of them, the price now exceeds their willingness to pay, and they will not purchase a shirt when they would have with import competition. (Also, to acknowledge Perot's point, there will be more domestic jobs in shirt production and fewer jobs in the exporting countries). The deadweight loss will be the area between the domestic supply curve, the total supply curve and the demand curve: $\frac{1}{2}hb = \frac{1}{2} (10-8)(70-40) = \30 (see separate graph). Note that this area does not look like the typical DWL triangle.

- g. Answer part (f) for the long-run.

In the long run, imports will not offer any competition to domestic producers, so there will be no change due to Perot's proposal. Hence, no winners, no losers and no deadweight loss.

7. Suppose that in NYC the daily demand for taxi rides is $Q = 2100 - 100P$ where P is the price in \$. Suppose also that the daily cost of operating each cab is a fixed \$100 dollar rental cost per vehicle, plus a variable cost of $q^2/100$, where q is the number of cab rides per cab per day.

(a) What is the long run total cost function of each cab? **LTC = 100 + $q^2/100$ per day.**

(b) If the market is a constant cost competitive one, what is the long-run price of a cab ride?

Minimum cost: $MC=LAC$, $q/50 = 100/q + q/100$, solved to find $q = 100$. Then $p = MC = 100/50 = 2$.

What is the number of rides each cab supplies and the number of cabs operating?

Found above that $q=100$. Then $Q = D(2) = 2100 - 200 = 1900$, and $n = Q/q =$

19.

(c) What is consumer surplus and producer surplus in the taxi cab market?

Inverse demand function: $P = 21 - Q/100$

Consumer Surplus: $\frac{1}{2}hb = \frac{1}{2} (21-2)(1900) = \$18,050$

Producer Surplus: \$0 (horizontal supply curve)

(d) Does the market achieve the condition for efficiency that $p=mc$. Explain.

As above with part (b), we have assumed that in the long run $p=MC$. Another way of looking at this is to note that in a competitive market, MC defines the supply curve of every firm, and therefore price has to equal marginal cost.

(e) Suppose that a tax of \$3 per ride is imposed. What is the new market price?

There is no change in the price received by the cabs (after taxes are passed back to the government) because this is determined by minimum LAC and remains at \$2. The effective price for consumers is then $\$5 = \$2 + \$3$.

What is number of rides per day, and number of cabs?

Then the number of rides per day is $Q = D(5)=1,600$, and the number of cabs is $Q/q = 16$.

(f) What is the deadweight cost of the tax per day? $\frac{1}{2}hb = \frac{1}{2} (3)(1,900-1,600) = \450

Monopoly, Price Discrimination

8. Kurt Vile produces and distributes the Libertarian Magazine, "Anarchy." Demand is given by $P = 55 - 2Q$. His cost function is $TC = 100 - 5Q + Q^2$.

a. What is Kurt's marginal revenue as a function of Q ? $MR = 55 - 4Q$

b. If Kurt wants to maximize profits, what price does he charge?

$$MC=MR: -5 + 2Q = 55 - 4Q, 6Q = 60, Q = 10, p = 55 - 2(10) = 35$$

How much profit and consumer surplus is generated at this price?

$$\text{Profit: } (35)(10) - (100 - 5(10) + 100) = 350 - 150 = \$200$$

$$\text{Consumer Surplus: } \frac{1}{2}hb = \frac{1}{2} (55-35)(10) = \$100$$

c. If Kurt wants to maximize total social surplus what price does he charge?

$$P=MC, \text{ intersection of demand and MC: } 55-2Q = -5+2Q, 60=4Q, Q=15, \\ p=55-2(15)= 25.$$

$$\text{What are his profits at this price? } (25)(15) - (100-5(15)+225) = 375 - 250 = \$125$$

d. What is the deadweight loss if profits are maximized?

$$MC(10) = -5+20 = 15, \quad DWL = \frac{1}{2}hb = \frac{1}{2} (35-15)(15-10) = \$50$$

9. Suppose the distributor charges Holiday Cinemas \$4 per ticket sold to rent the movie, "STAR WARS 15: THE NEXT BILLION." Suppose the theater can seat a maximum of 200 people. Suppose also that the demand to see the movie is given by $P = 10 - Q/10$ in the afternoon, and $P = 20 - Q/10$ in the evening.

(a) Calculate the profit maximizing price in the evening and the afternoon, and the number of people who see each show. (Using graphs to set up the problem will be helpful)

$$\text{Afternoon: } MC=\$4, MR = 10 - Q/5, MR=MC: 6=Q/5, Q=30, p = 10 - 30/10 = 7. \\ \text{Evening: } MC=\$4, MR = 20 - Q/5, MR=MC: 16=Q/5, Q=80, p = 20 - 80/10 = 12.$$

(b) What is the amount of revenue paid to the movie distributor? $4(30+80) = \$440$
(Total Profit: $(30)(7) + (80)(12) - 440 = 210 + 960 - 440 = \730)

(c) Suppose that the distributor instead asks the theater owner for a flat fee of \$1000 to show the movie, with no charge per customer. Calculate whether or not the theatre owner would prefer this arrangement.

$$\text{Afternoon: } MC=\$0, MR = 10 - Q/5, MR=MC: Q=50, p=10-5=5, Rev = \$250 \\ \text{Evening: } MC=\$0, MR = 20 - Q/5, MR=MC: Q=100, p=20-10=10, Rev = \$1,000 \\ \text{Profit: } \$250 + \$1,000 - \$1,000 = \$250, \text{ the theatre owner would } \underline{\text{not prefer}} \text{ this arrangement.}$$

(d) Show whether the fee per viewer of \$4 charged by the distributor, or the flat fee of \$1000 results in a more EFFICIENT outcome.

1st Arrangement:

$$\text{Consumer Surplus: } \frac{1}{2}hb \text{ afternoon \& evening: } \frac{1}{2} (10-7)(30) + \frac{1}{2} (20-12)(80) = \\ 45+320 = \$365.$$

$$\text{Producer Surplus: (With constant MC, producer surplus is the rectangle with} \\ \text{height equal to the difference between } p \text{ and MC, and base equal to } Q.) \\ \text{hb afternoon \& evening: } (7-4)(30) + (12-4)(80) = 90 + 640 = \$730.$$

$$\text{Total Surplus: } 365 + 730 = \$1,095.$$

2nd Arrangement:

Consumer Surplus: $\frac{1}{2}$ hb afternoon & evening: $\frac{1}{2} (10-5)(50) + \frac{1}{2} (20-10)(100) = 125+500 = \625 .

Producer Surplus: hb afternoon & evening: $(5-0)(50) + (10-0)(100) = 250 + 1,000 = \$1,250$.

Total Surplus: $625 + 1,250 = \$1,875$.

Therefore, the 2nd Arrangement is more efficient (with the flat fee).

- (e) What is the efficient price for admission in the afternoon and in the evening.

The efficient price will depend on the arrangement:

With 1st: $MC=P$, $P=4$ both afternoon & evening (with $Q=60$ & $Q=160$).

With 2nd: $MC=P$, $P=0$ both afternoon & evening (with $Q=100$ & $Q=200$).

10. Auto dealerships are of two sorts. Those which bargain over prices, and those which sell only at the posted price. Customers report great dissatisfaction with dealers that bargain over prices. Yet the number of fixed price dealerships has been declining recently. Explain why.

With fixed price dealerships, all buyers of the same car model will pay the same price, irrespective of their willingness to pay, as long they are willing to pay at least as much as the sticker price. However with bargaining, the dealership can post a higher sticker price and then bargain. If the dealership is good at this, they can peg the actual selling price closer to the buyer's willingness to pay and thus capture more of the buyer's consumer surplus.

11. Suppose that the demand for electricity by residential consumers is the same for all consumers and is given by $P = 1 - Q/100$ where P is the price per kw-hour in \$ and Q is the kw-hours per week. Suppose that the marginal cost of supplying electricity is \$0.20 per kw-hour.

- (a) If the electricity company can charge a fixed fee as well as a price per unit, calculate the profit maximizing fee per week, the quantity of units sold, and the price charged.

The electric company will charge a price per unit (p) equal to its marginal cost and then set the fixed fee (F) so that it captures all consumer surplus. Then in this case $MC = p = \$0.20$, so that quantity may be found with the demand function: $0.20 = 1 - Q/100$, $Q = 80$ kw-hours per week. With only this simple price, $CS = \frac{1}{2}hb = \frac{1}{2}(1-0.2)(80) = \32 . Thus $K = \$32$ per week.

- (b) What is the consumer surplus under the profit maximizing pricing scheme?

By construction, there is zero consumer surplus (after subtracting K).

- (c) Is the profit maximizing pricing scheme socially efficient? Explain.

This arrangement can be shown to be efficient in two ways. First, $p=MC$, that is the incremental payment for the last unit consumed equals the incremental cost of creating that last unit of output. Second, total surplus is maximized, even though all of this surplus is appropriated by the producer.

12. Suppose that it is proposed to run a train between Sacramento and San Francisco. The smallest train seats 150 people and the cost per trip is found to be \$2,500, irrespective of how many people travel. The demand for travel (in \$) is $P = 40 - Q/5$.

(a) On a diagram draw the demand curve for service each day and the marginal cost curve.

The marginal cost per passenger is \$0 up till 150 passengers, then it rises steeply since a new train has to be added. See separate page for graph.

(b) What will happen if a private firm tries to provide the service and can charge only one price? Explain.

The private firm will try to maximize profits by setting $MC=MR$: $0 = 40 - Q/(2.5)$, $Q = 100$, $p = 40 - 20 = 20$, and "profit" is $(20)(100) - 2,500 = -\$500$. Thus, at best the firm will operate at a loss. *So the private firm will not offer service.*

(c) Show what the socially efficient price for the service is, and that it is socially beneficial to provide the service.

The socially efficient price is to set $p = MC = \$0$ up till the train is full. After that the price has to be set to ration the demand so that exactly 150 people want to use the train. So $P = 40 - (150)/5 = 10$. At a price of 10, the total surplus from running the train (CS+PS) is $\frac{1}{2} (40-10)(150) + 10 \cdot 150 = 3,750$. Since total surplus is larger than the cost, it is socially beneficial to provide this service.

(d) If the private firm was able to effectively price discriminate would it provide the service and would it provide it at efficient levels? Explain.

With perfect price discrimination, the private firm could capture all the consumer surplus and then obtain a profit of $3,750 - 2,500 = \$1,250$. So it would operate the service.