

Final Exam - Answer Key

Directions: Answer all questions; points for each question are given in parentheses. For full credit, you must provide complete explanations for your answers.

1. (5) Timothy Geithner is seeking your advice on assessing the interest rate risk of some of the troubled banks that are currently undergoing a "stress" test. Geithner's assistant is arguing that the FGAP is the most relevant measure. Do you agree?

Answer: No, I don't agree. The most important measure of a firm's financial health is its net worth. Funding Gap is important for determining the exposure of net interest income to interest rate changes but ignores the interest rate exposure of long term assets and liabilities. Hence, Duration Gap is a more appropriate measure since this measures the exposure of the entire balance sheet, i.e. net worth, to interest rate risk.

2. (10) Use the expectations hypothesis to answer the following questions:

- (a) The yield curve typically has a positive slope. What does this imply about the predicted behavior of short-term interest rates? Does this suggest a problem with the expectations hypothesis?

Answer: A positively sloped yield curve implies, according to the expectations hypothesis, that short term interest rates are going to rise. But, since the yield curve is usually upward sloping, this would imply that short term interest rates would always be rising, which is not observed. This provides evidence against the expectations hypothesis and suggests that investors are compensated for holding longer maturity bonds - i.e. there is a positive term premium.

- (b) If the interest rates on one- to five-year bonds are currently 4%, 5%, 6%, 7% and 8%, predict what the one-year interest rate will be two years from now?

Answer: All you need for this are the yields on 2- and 3-year bonds: According to the expectations hypothesis, $(1 + i3_t)^3 = (1 + i2_t)^2 (1 + i1_{t+2}^e)$. Or, using a log approximation: $3i3_t = 2i2_t + i1_{t+2}^e$. Using the numbers yields: $i_{t+2}^e = 8\%$.

3. (5) In Brunnermeier's analysis of the credit crisis, the TED spread played a critical role. Why?

Answer: The TED spread is the difference between LIBOR (London interbank offered rate) and the rate on a Treasury bill. This captures the risk and liquidity associated with the interbank loan market. During the course of the financial crisis, the TED spread spiked at various times as banks liquidity needs increased. For instance, banks had lines of credit issued to their subsidiaries which held CDOs; as the value of these came into question, the subsidiaries could not borrow on the open market (issue commercial paper or repurchase agreements). Consequently, they tapped their lines of credit and this increased the demand for liquidity by banks. This caused a jump in the TED spread.

4. (15) In the model of banks as providers of liquidity insurance, explain **precisely** why the existence of banks can yield a Pareto optimum. Why is the *Law of Large Numbers* relevant?

Answer: The existence of banks can produce a Pareto optimal allocation since the banks can offer full and fairly priced liquidity insurance. Let N be the number of households where N is large. Each household has the probability of π_1 that they are early consumers; if they are early consumers, their optimal consumption is C_1 . This implies that the expected amount of consumption in the first period is $\pi_1 C_1$. With N very large, the amount of consumption in the first period is exactly equal to $\pi_1 C_1$. This is an implication of the *Law of Large Numbers*.

5. (20) One of the implications of the financial accelerator model (i.e. the model presented in the article by Gertler and Hubbard) is that investment may behave asymmetrically over the business cycle. Explain why this is the case. A good answer will be supported by graphical analysis similar to that presented in the article.

Answer: Equilibrium in the financial accelerator model is characterized by two key conditions: firms are maximizing expected profits and the incentive compatibility constraint is satisfied. When firms' net worth is large, the incentive compatibility constraint is a weak inequality and so the only determinant of investment is the market interest rate. So increases in net worth will not change optimal investment in this case. As net worth falls, as in a recession, then the incentive compatibility constraint will become binding. In this case, net worth and the market interest rate determine investment. A further fall in net worth will lower investment even if interest rates do not change. A graph similar to Figure 2 in the article is necessary for full credit.

6. (15) Suppose agents derive utility from income as given by the function: $U(y) = \ln y$. Answer the following:

- (a) Prove that this function implies that agents' measure of relative risk aversion is constant. What is it equal to?

Answer: Relative risk aversion is measured as: $\gamma = -\frac{U''}{U'}y$. For $\ln y$, we have: $U' = y^{-1}$, $U'' = -y^{-2}$. So $\gamma = 1$.

- (b) Suppose agents are faced with two possible income levels: $y_1 = 100$ and $y_2 = 50$ and assume that the probability of the low income state is 20%. If actuarially fair insurance was offered to an agent, what would the premium for insurance cost (assuming zero expected profits for insurance companies).

Answer: In actuarially fair insurance is offered, this implies insurance companies are making zero profits. Let h = insurance premium, $p = 0.20$ = the probability of the low income state and $L = \text{loss}$ (equal to 50). We know that risk averse agents will purchase full insurance so the expected profits of the insurance company is: $E(\pi) = h - pL = 0$ so that $h = 0.20(50) = 10$.

- (c) Show that the utility of agents from purchasing insurance is greater than the expected utility from no insurance (using the functional form for utility given above).

Answer: With full insurance, the agent has the same income in both states. In the high income state, $Y = 100 - h = 90$. In the low income state: $Y = 50 - h + L = 50 - 10 + 50 = 90$. So we need to compare $U(90)$ to $E(U(y))$. $U(90)$ is given by $\ln(90) = 4.499$; $E(U) = 0.8 * \ln(100) + 0.20 * \ln(50) = 4.466$.

7. (20) In discussing the modern version of the IS curve, we showed that the linearized Euler equation has the form:

$$\tilde{c}_t = E_t [\tilde{c}_{t+1}] - \sigma \tilde{r}_t$$

where c_t is consumption and rr_t is the **one-period** real interest rate (the tildes imply that the terms are expressed as deviations from full employment levels). In their article, Galí and Gertler stress that it is the **long term** real interest rate that is critical for aggregate demand movements. How do you reconcile Galí and Gertler's statement with the above representation of the Euler equation?

Answer: It is possible to recursively substitute out future values of \tilde{c}_{t+i} using the linearized Euler equation to obtain:

$$\tilde{c}_t = -\sigma E_t \left[\sum_{i=0}^{\infty} \tilde{r}_{t+i} \right]$$

That is, it is the entire path of short term interest rates that affects current consumption decisions. But, according to the expectations hypothesis, the long term interest rate is equal to the summation term (when interest rates are measured in deviation form). The intuition is straightforward: today's consumption is affected by today's short term interest rate and expected consumption next period. But consumption next period will be affected by next period's short term interest rate and expected consumption in period $t + 2$. And this is true for all future periods. The path of expected short term interest rates, in turn, determines the long-term interest rate.

8. (10) The Taylor rule is often expressed as:

$$r_t = \pi_t + \alpha (\pi_t - \pi^*) + \beta (y_t - \bar{y}_t) + rr$$

where r_t is the short term interest rate (i.e. the Fed Funds rate), π_t is the current inflation rate, π_t^* is the target inflation rate chosen by the monetary authorities (i.e. the Fed), y_t is GDP, \bar{y}_t is full employment GDP, and rr is the real interest rate consistent with full employment. Explain why a value for $\alpha < 0$ is considered problematic for stable monetary policy.

Answer: If $\alpha < 0$, then the nominal interest rate increases less than one-for-one with a change in the inflation rate. (The coefficient on inflation is $(1 + \alpha)$). Since the real interest rate (ex-post) is $r_t - \pi_t$, this implies that the real interest rate would fall with an increase in inflation. But, as seen in Q7 above, a lower real interest rate would stimulate consumption demand and lead to increased output. If the aggregate supply curve is upward sloping, this would imply an increase in inflation and, due to the Taylor rule, the real interest rate would fall further. So the economy is on an accelerating inflation path. This is why economist's argue that $\alpha > 0$ is required for stable policy.