CHAPTER

1

Basic Concepts

"Practical men, who believe themselves to be quite exempt from any intellectual influences, are usually the slaves of some defunct economist. Madmen in authority, who hear voices in the air, are distilling their frenzy from some academic scribbler of a few years back. I am sure that the power of vested interests is vastly exaggerated compared with the gradual encroachment of ideas."

John Maynard Keynes: *The General Theory of Employment, Interest and Money*, 1947.

INTRODUCTION

The modern theory of financial intermediation is based on concepts developed in financial economics. These concepts are used liberally throughout the book, so it is important to understand them well. It may not be obvious at the outset why a particular concept is needed to understand banking. For example, some may question the relevance of "market completeness" to commercial banking. Yet, this seemingly abstract concept is of great relevance to understanding financial innovation, securitization, and the off-balance sheet activities of banks. Many other concepts such as riskless arbitrage, options, market efficiency, and so on have long dominated other subfields of finance and are transparently of great significance for a study of banking. What is common to all these concepts is that they are not banking concepts *per se,* but are useful in comprehending banking practices. We have thus chosen to consolidate these concepts in this chapter, to provide easy reference for those who may be unfamiliar with them.

RISK PREFERENCES

To understand the economic behavior of individuals, it is convenient to think of an individual as being described by a utility function that summarizes preferences over different outcomes. For a wealth level W, let U(W) represent the individual's utility of that wealth. It is reasonable to suppose that this individual always prefers more wealth to less. This is called "non-satiation" and can be expressed as $U'(W) > 0$, where the prime denotes a derivative. That is, at the margin, an additional unit of wealth always increases utility by some amount, however small.

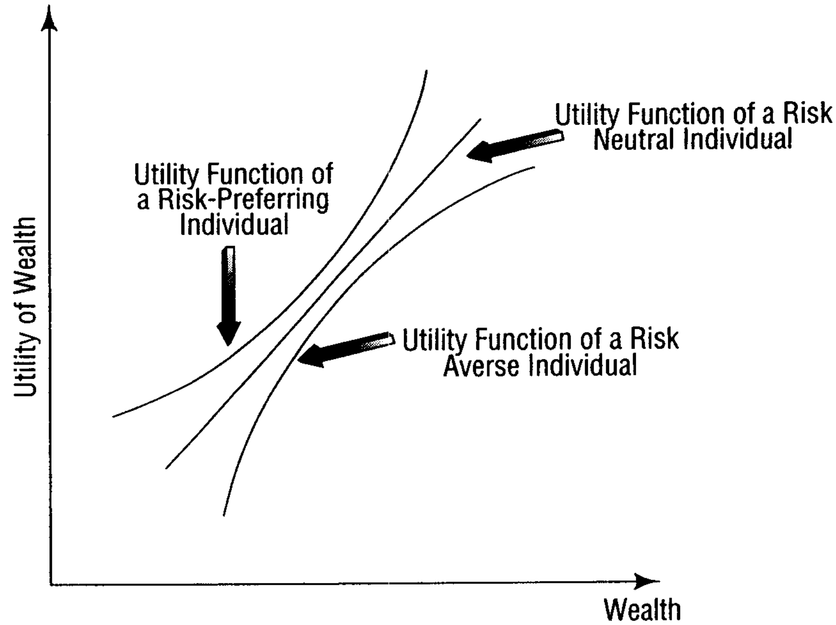
An individual can usually be classified as being either risk neutral, risk averse or risk-preferring. If *risk neutral,* he is indifferent between the certainty of receiving the expected value of a gamble and the uncertainty of the gamble itself. That is, he cares only about his expected wealth and not about variability *per se.* For a risk neutral individual, $U''(W) = 0$, that is, his utility function is *linear* in wealth. Letting $E(\bullet)$ denote the statistical expectation operator, we can write $U[E(W)] = EU(W)$ for a risk neutral individual, where $U[E(W)]$ is the utility of the expected value of W and $EU(W)$ is the expected utility of W. For such an individual, changing the risk of an outcome has no effect on his well-being so long as the expected outcome is left unchanged.

The utility function of a *risk averse* individual is *concave* in wealth, that is, $U''(W) < 0$. Such an individual prefers a certain amount to a gamble with the same expected value. *Jensen's inequality* says that

$$U\big[E(W)\big] > E\big[U(W)\big]$$

if U is (strictly) concave in W. Thus, risk averse individuals prefer less risk to more, or equivalently, they demand a premium for being exposed to risk.

FIGURE 1.1 **Three Different Types of Utility Functions**



A *risk-preferring* individual will always prefer the riskier of two outcomes having the same expected value. The utility function of a risk-preferring individual is *convex* in wealth, that is, $U''(W) > 0$. Jensen's inequality says that

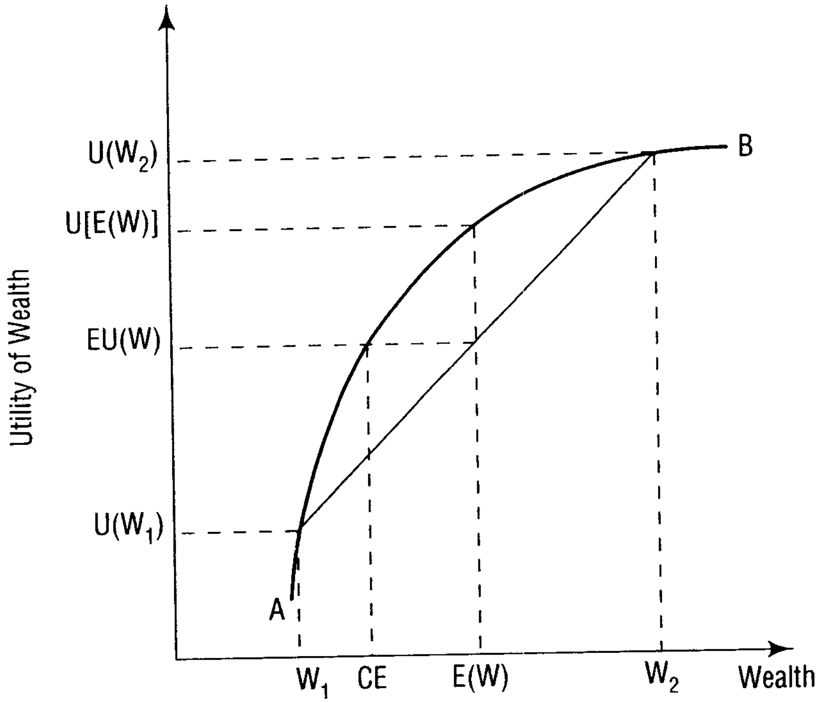
$$U\big[E(W)\big] < E\big[U(W)\big]$$

if U is (strictly) convex in W.

Despite the popularity of lotteries and pari-mutuel betting, it is commonly assumed that individuals are risk averse. Most of finance theory is built on this assumption. In Figure 1.2 we have drawn a picture to indicate what is going on.

Consider a gamble in which an individual's wealth W can be either W_1 with probability 0.5 or W_2 with probability 0.5. If the individual is risk averse, he has a concave utility function that may look like the curve AB. Now, his expected wealth from the gamble is $E(W) = 0.5W_1 + 0.5W_2$, which is precisely midway between W_1 and W_2. The utility derived from receiving this expected wealth is given by $U[E(W)]$ on the y-axis. However, if this individual accepts the gamble itself [with an expected value of $E(W)$], then his expected utility, $EU(W)$, is midway between $U(W_1)$ and $U(W_2)$ on the y-axis, and can be read off the vertical axis as the point of intersection between the vertical line rising from the midpoint between W_1 and W_2 on the x-axis and the straight line connecting $U(W_1)$ and $U(W_2)$. As is clear from the picture, $U[E(W)] > EU(W)$. The more bowed or concave the individual's utility function, the more risk averse that individual will be and the larger will be the difference between $U[E(W)]$ and $EU(W)$.

FIGURE 1.2 **Risk Aversion and Certainty Equivalent**



We can also ask what sure payment we would have to offer this risk averse individual to make him indifferent between that sure payment and the gamble. Such a sure payment is known as the *certainty equivalent* of the gamble. In the figure, this certainty equivalent is denoted by CE on the x-axis. Since the individual is risk averse, the certainty equivalent of the gamble is less than its expected value. Alternatively expressed, E(W) – CE is the *risk premium* that the risk averse individual requires in order to participate in the gamble if his alternative is to receive CE for sure.

The concept of risk aversion is used frequently in this book. For example, we use it in Chapter 3 to discuss the role of financial intermediaries in the economy, and then again in Chapter 4 when we discuss liquidity premiums embedded in the term structure of interest rates. Risk aversion is also important in understanding financial innovation, deposit insurance, and a host of other issues.

DIVERSIFICATION

We have just seen that risk averse individuals prefer to reduce risk. One way to reduce risk is to diversify. The basic idea behind diversification is that if you hold numerous risky assets, your return will be more predictable, but not necessarily greater. For diversification to work, it is necessary that returns on the assets in your portfolio not be perfectly and positively correlated. Indeed, if they are so correlated, the assets are identical for practical purposes so that the opportunity to diversify is defeated. Note that risk can be classified as idiosyncratic or systematic. An idiosyncratic risk is one that stems from forces specific to the asset in question, whereas systematic risk arises from the correlation of the asset's payoff to economywide phenomena such as depression. Idiosyncratic risks are diversifiable, systematic risks are not.

To see how diversification works, suppose that you hold two assets, A and B, whose returns are random variables.[1] Let the variances of these returns be σ_A^2 and σ_B^2, respectively. Suppose the returns on A and B are perfectly and positively correlated, so that $\rho_{AB} = 1$, where ρ_{AB} is the correlation coefficient between A and B. The proportions of the portfolio's value invested in A and B are y_A and y_B, respectively. Then the variance of your portfolio return is

$$\sigma_p^2 = y_A^2\sigma_A^2 + 2y_Ay_B \operatorname{Cov}(A,B) + y_B^2\sigma_B^2 \tag{1.1}$$

where Cov(A,B) is the covariance between the returns on A and B. Then, using

$$\operatorname{Cov}(A,B) = \rho_{AB}\sigma_A\sigma_B \tag{1.2}$$

we have

$$\sigma_p^2 = y_A^2\sigma_A^2 + 2y_Ay_B\, \rho_{AB}\sigma_A\sigma_B + y_B^2\sigma_B^2 \tag{1.3}$$

Since $\rho_{AB} = 1$, the right-hand size of (1.3) is a perfect square, $(y_A\sigma_A + y_B\sigma_B)^2$. As long as $y_A\sigma_A + y_B\sigma_B \geq 0$, we can write (1.3) as

$$\sigma_p = y_A\sigma_A + y_B\sigma_B. \tag{1.4}$$

Thus, if $\rho_{AB} = 1$, the standard deviation of the portfolio return is just the weighted average of the standard deviations of the returns on assets A and B. *Diversification*

---

1. Suppose x and z are two random variables that can each take any value from $-\infty$ to $+\infty$. A random variable is one whose behavior is described by a probability density function,but its precise value is unknown. Let f(x) and g(z) be the density functions of x and z, respectively. Then, the probability that x will lie between the two numbers a and b is $\Pr(a \leq x \leq b) = \int_a^b f(x)dx \geq 0$, and $\int_{-\infty}^{\infty} f(x)dx = 1$. The statistical mean (expected value) of x is $E(x) = \int_{-\infty}^{\infty} xf(x)dx$, its variance is $\sigma_x^2 = \int_{-\infty}^{\infty} [x - E(x)]^2 f(x)dx$, and the mean and variance of z are analogously defined. The covariance of x and z is $\operatorname{Cov}(x,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [x - E(x)][z - E(z)] f(x)g(z)dxdz$ and the correlation between x and z is $\rho_{xz} = \operatorname{Cov}(x,z)/\ \sigma_x\sigma_z$, where σ_x and σ_z are the standard deviations (square roots of the respective variances) of x and z, respectively.

therefore does not reduce portfolio risk when returns are perfectly and positively correlated. For any general correlation coefficient ρ_{AB}, we can write the portfolio return variance as

$$\sigma_p^2 = y_A^2\sigma_A^2 + 2y_Ay_B\rho_{AB}\sigma_A\sigma_B + y_B^2\sigma_B^2. \quad [1.5]$$

Holding fixed y_A, y_B, σ_A and σ_B, we see that $\partial\sigma_p^2/\partial\rho_{AB} > 0$, that is, portfolio risk increases with the correlation between the returns on the component assets. At $\rho_{AB} = 0$ (uncorrelated returns),

$$\sigma_p^2 = y_A^2\sigma_A^2 + y_B^2\sigma_B^2. \quad [1.6]$$

Example 1.1 To see that diversification helps in this case, suppose $y_A = y_B = 0.5$, $\sigma_A^2 = 100$, $\sigma_B^2 = 144$. Calculate the variance of a portfolio of assets A and B, assuming first that the returns of the individual assets are perfectly positively correlated, $\rho_{AB} = 1$, and then that they are uncorrelated, $\rho_{AB} = 0$.

Solution In the case of perfectly and positively correlated returns, $\sigma_p = 0.5\ (10) + 0.5\ (12) = 11$, or $\sigma_p^2 = 121$. With uncorrelated returns, (1.6) implies that $\sigma_p^2 = 0.25\ (100) + 0.25(144) = 61$. Thus, not only is this variance lower than with perfectly and positively correlated returns, but it is also lower than the variance on either of the component assets.

The maximum effect of diversification occurs when ρ_{AB} is at its minimum value of −1, that is, returns are perfectly negatively correlated. In this case

$$\sigma_p^2 = y_A^2\sigma_A^2 - 2y_Ay_B\sigma_A\sigma_B + y_B^2\sigma_B^2 \quad [1.7]$$

so that

$$\sigma_p = y_B\sigma_B - y_A\sigma_A. \quad [1.8]$$

This seems to indicate that the portfolio will have some risk, albeit lower than in the previous cases. But suppose we construct the portfolio so that the proportionate holdings of the assets are inversely related to their relative risks. That is,

$$y_A/y_B = \sigma_B/\sigma_A \quad [1.9]$$

or

$$y_A = \sigma_B y_B / \sigma_A. \tag{1.10}$$

Substituting (1.10) in (1.8) yields

$$\sigma_p = y_B\, \sigma_B - (\sigma_B y_B \sigma_A / \sigma_A) = 0$$

indicating that in this special case of perfectly negatively correlated returns, portfolio risk can be reduced to zero!

Even when assets with perfectly negatively correlated returns are unavailable, we can reduce portfolio risk by adding more assets (provided they are not perfectly positively correlated with those already in the portfolio), while keeping fixed the total wealth invested in the portfolio.[2] To illustrate, suppose we have N assets available, each with returns pairwise uncorrelated with the returns of every other asset. In this case, a generalized version of (1.6) is

$$\sigma_p^{\ 2} = \sum_{i=1}^{N} y_i^{\ 2} \sigma_i^{\ 2} \tag{1.11}$$

where y_i is the fraction of the portfolio value invested in asset i, where i = 1, . . . , N, and σ_i^2 is the variance of asset i. Suppose we choose $y_i = 1/N$.

Then, defining σ^2_{max} as the maximum variance among the σ_i^2 (we assume $\sigma^2_{max} < \infty$, and permit $\sigma_i^2 = \sigma^2$ for all i in which case $\sigma^2_{max} = \sigma^2$), (1.11) becomes

$$\begin{aligned}\sigma_p^2 &= \sum_{i=1}^{N} \left[\frac{1}{N}\right]^2 \sigma_i^2 \\ &\leq N\left[\frac{1}{N}\right]^2 \sigma^2_{max} \\ &= \frac{\sigma^2_{max}}{N}\end{aligned}$$

As N increases, σ_p^2 diminishes. In fact, in the limit as N goes to infinity, $\sigma_p^{\ 2}$ goes to zero. Thus, if we have sufficiently many assets with (pairwise) uncorrelated returns, we can drive portfolio risk as low as we wish and make returns predictable to any desired accuracy.

---

2. In pointing out the "fallacy of large numbers," Samuelson (1963) shows that diversification is not necessarily preferred by a risk averse individual if one also adds more wealth to the portfolio as more assets are added. We will have more to say about this in Chapter 3.

An obvious question is why investors do not drive their risks to zero. First, not all risks are diversifiable. Some contingencies affect all assets alike and consequently holding more assets will not alter the underlying uncertainty. This is the notion of *force majeure* in insurance. Natural calamities such as floods and earthquakes are examples, as are losses attributed to wars. Second, as the investor increases the number of securities held in the portfolio, there are obvious costs of administration. These costs are sufficient to restrain diversification, but in addition numerous studies indicate that a large fraction of the potential benefits of diversification are obtained by holding a relatively small number of securities. That is, the marginal benefits of diversification decline rapidly as the number of securities increases.

Finally, cross-sectional reusability of information restrains the incentive to diversify. We shall have more to say in Chapter 3 about information reusability since this is a major force motivating the emergence of financial intermediaries. Suffice to say that if a lender invests to learn about a customer in the steel business in order to make a loan, it will see a potential benefit to lending to others in the steel business because this will spread the cost of learning about the first steel industry borrower. This conserves on the costs of becoming informed, which is a special type of transactions cost. This inclination restrains diversification. Thus, we see diversification within areas of specialization among most financial intermediaries. And when we speak of financial intermediaries processing risk, we mean that they are usually simultaneously diversifying some, absorbing some, and shifting some to others.

The concept of diversification is used in this book in a variety of different contexts. We use it quite extensively in Chapter 3 to explain how it can lead to natural economies of scale in the production of financial intermediation services and in Chapter 4 to explain how banks can use diversification to reduce the risks they face. It is also a very useful concept in understanding securitization and deposit insurance.

RISKLESS ARBITRAGE

Arbitrage is the simultaneous purchase and sale of given goods or securities that are trading at disparate prices. This opportunity for riskless profit is transitory because the exploitation of such opportunities eliminates the initial price disparities.

The term *arbitrage* is often loosely applied to situations in which the objects of trade are similar, but not identical, and where the risk is thought to be small, but not totally absent. Since such situations are often referred to as arbitrage, the redundant "riskless arbitrage" has emerged to describe arbitrage rather than limited risk speculation (a situation in which a profit can be had for a small amount of risk). Thus, succinctly defined, riskless arbitrage is profit without risk and without investment. We shall later discuss "risk-controlled arbitrage" as an illustration of limited risk speculation. Consider the following illustration of riskless arbitrage.