

## Assessing Interest Rate Risk: The Funding Gap and Duration Gap Analysis

We focus on ways to measure interest rate risk exposure of banks.

There are two main views:

1. Funding Gap: this is the income view. How will interest rate fluctuations affect NII?
2. Duration Gap: this is the balance sheet view. How will interest rate fluctuations affect equity?

Conclusion: both are important. Can not insulate both risks simultaneously.

## Funding Gap – Managing Interest Income in the short term

Implementation is simple:

1. Divide up assets/liabilities into maturity buckets
2. Within each bucket, determine the value of rate sensitive assets (RSA) and value of rate sensitive liabilities (RSL).
3. Then the funding gap is simply:

$$FGAP = \text{Value of RSA} - \text{Value of RSL}$$

## Funding Gap – Managing Interest Income in the short term

Why is this useful? Recall definition of NII:

$$NII = FRA \cdot r_1 + RSA \cdot r_2 - FRL \cdot i_1 - RSL \cdot i_2$$

Where  $FRA$  = fixed rate assets,  $FRL$  = fixed rate liabilities. Then, the change in  $NII$  can be written as (where I assume  $\Delta r_2 = \Delta i_2 = \Delta i$ ):

$$\Delta NII = RSA \cdot \Delta r_2 - RSL \cdot \Delta i_2 = (RSA - RSL) \Delta i = FGAP \cdot \Delta i$$

Note that  $\text{Sign}[Cov(\Delta NII, \Delta i)] = \text{Sign}[FGAP]$

Where  $Cov$  = covariance.

## Funding Gap – Example

Consider the following simple example:

Balance Sheet of Southern Rock Bank

	Assets	Yield	Liabilities	Yield
Rate Sensitive	500	12%	600	9%
Fixed Rate	350	15%	220	8%
			Equity=30	
Total	850		850	

$NII = \$40.90$ ,  $NIM = 4.81\%$ ,  $FGAP = -100$ .

Suppose  $\Delta i = +1\%$  then  $NII = \$39.90$ .

### Funding Gap and NIM variation

The relationship between  $FGAP$  and  $NIM$  can be used to accept an acceptable variation in  $NIM$  for a given forecast in interest rate variation.

Note that:

$$FGAP \cdot \Delta i = \Delta NII = \left( \frac{\Delta NII}{Assets} \right) (Assets) = \left( \frac{\Delta NIM}{NIM} \right) (NIM) (Assets)$$

Re-write this as:

$$\frac{\text{Target Gap}}{Assets} = \frac{(\% \Delta NIM)(NIM)}{\Delta i}$$

### Funding Gap – Managing Interest Income in the short term

$$\frac{\text{Target Gap}}{Assets} = \frac{(\% \Delta NIM)(NIM)}{\Delta i}$$

Suppose a bank has \$50 million in assets and wants a  $NIM$  of 5%. It is willing to accept a variation of  $NIM$  of 20%. If it forecasts that interest rate volatility of 4% is possible, what is the acceptable  $FGAP$ .

$$\frac{\text{Target Gap}}{Assets} = \frac{(20)(5)}{4} = 25\%$$

Given assets of \$50 million, this implies a  $FGAP = 12.5$  million (could be positive or negative).

### Funding Gap – Managing Interest Income in the short term

Funding Gap concerns:

1. Arbitrary to decide what is fixed versus variable rate.
2. Main problem: ignores the effect that changes in interest rates have on bank's net worth due to change in value of fixed rate assets and liabilities.

Use Duration Gap to measure (2).

### Duration Gap

Duration Gap is the difference between the average duration of assets and the average duration of liabilities.

Main Street Bank's Assets

	Value	Rate	Duration
Cash	100	0	0
3-yr loan	700	14%	2.65
7-yr T-Bond	200	12%	5.97
	1000		3.05

$$\bar{D}_A = \sum_{i=1}^N \left( \frac{MV_i}{Assets} \right) D_i$$

$$\bar{D}_A = \left( \frac{700}{1000} \right) (2.65) + \left( \frac{200}{1000} \right) (5.97) = 3.05 \text{ years}$$

## Duration Gap

Duration Gap is the difference between the average duration of assets and the average duration of liabilities.

### Main Street Bank's Liabilities

	Value	Rate	Duration
1-yr Time Deposit	520	9%	1
4-yr CD	400	10%	3.49
Total Liabilities	920		
Equity	80		
Total	1000		1.92

$$\bar{D}_L = \sum_{i=1}^N \left( \frac{MV_i}{Assets} \right) D_i$$

$$\bar{D}_L = \left( \frac{520}{1000} \right) (1.00) + \left( \frac{400}{1000} \right) (3.49) = 1.92 \text{ years}$$

## Duration Gap

Duration Gap is the difference between the average duration of assets and the average duration of liabilities.

$$DGAP = \bar{D}_A - \bar{D}_L = 3.05 - 1.92 = 1.13 \text{ years}$$

For Main Street Bank, if interest rates rise, Net Worth falls and vice versa.

In general:

$$Cov(NW, i) \gtrless 0 \text{ as } DGAP \lesseqgtr 0$$

More specifically:

$$\% \Delta Equity = -DGAP \times \frac{\Delta i}{1+i}$$

## Duration Gap

Duration Gap concerns:

1. Requires calculation of duration of all assets and liabilities – this requires much information: each account's interest rate information, prepayment options, default probabilities, etc.
2. Need estimates of changes in interest rates of different maturities and risk.
3. Duration changes over time and at different rates for different securities – Duration drift. So in practice it is necessary to recalculate DGAP frequently.

## Final topic: Impossible to manage FGAP and DGAP simultaneously

It is not possible to simultaneously immunize against interest rate risk as implied by FGAP and DGAP.

To demonstrate this, first note that the equity of a firm is given by the present discounted value of dividends. If a firm is infinitely lived and pays a constant fraction  $\delta$  of NII as dividends, then equity is given by the price of a consol (note that NII is written as a function of interest rates)

$$E = \frac{\delta \times NII(i)}{i}$$

Take the derivative with respect to interest rates:

$$\frac{dE}{di} = \frac{\delta}{i} \frac{dNII}{di} - \frac{\delta \times NII(i)}{i^2} = \frac{\delta}{i} \frac{dNII}{di} - \frac{E}{i}$$

Express this in discrete change form, i.e.  $di \approx \Delta i$

$$\Delta E = \frac{\delta \times \Delta NII}{i} - \frac{E}{i} \Delta i$$

Final topic: Impossible to manage FGAP and DGAP simultaneously

Rewriting:

$$\Delta E = \frac{\delta \times \Delta NII}{i} - \frac{E}{i} \Delta i$$

Recall that  $\Delta NII = FGAP \cdot \Delta i$

So, if  $FGAP = 0$ , then equity will fall (due to increase in discount factor).

To immunize equity, then  $FGAP$  must be positive.

For practical purposes, probably best to focus on equity (DGAP).

Hedging Interest Rate Risk

This quick overview presented some quantitative measures of banks' exposure to interest rate risk.

The fact is that most banks (of financial intermediaries) will have, by the very nature of their business a positive DGAP.

What to do?

Hedge interest rate risk using interest rate futures.