Uncertainty and the Effectiveness of Policy

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OF POLICY*

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Economists concerned with aggregative policy spend a great deal of their time discussing the implications of various structural changes for the effectiveness of economic policy. In recent years, for example, monetary economists have debated at great length whether the rapid growth of nonbank financial intermediaries has lessened the effectiveness of conventional instruments of monetary control. Similarly, in discussions of the desirability of the addition or removal of specific financial regulations the consequences for the effectiveness of policy play an important role. One of the striking features of many of these discussions is the absence of any clear notion of what "effectiveness" is. At times it appears to be simply "bang per buck"—how large a change in some crucial variable (e.g., the long-term bond rate) results from a given change in a policy variable (e.g., open market operation). A natural question to ask is why a halving of effectiveness in this sense should not be met simply by doubling the dose of policy, with equivalent results.

It seems reasonable to suppose that the consequences of a structural change for the effectiveness of policy should be related to how it affects the policy-maker's performance in meeting his objectives. Suppose, for example, that the policy-maker wants to maximize a utility function which depends on the values of "target" variables. If, after some structural change, the policy-maker finds he is able to score higher on his utility function, then presumably the structural change has improved the effectiveness of policy and vice versa. One of the implications of the "theory of policy" in a world of certainty [6] or "certainty equivalence" [1] [3] [4] [5] is that structural changes which simply alter the magnitude of the response to policy do not alter the attainable utility level.[1] Hence such structural changes do not alter effectiveness in the above sense. Another feature of the theory of policy in a world of certainty is that a policy-maker with more instruments than targets is free to discard the excess instruments, and it makes no difference to his performance which ones he discards. These results are crucially dependent on the assumption that the response of target variables to policy in-

[1] Assuming that the levels of instruments do not enter directly into the utility function.

struments is known for certain. Since it is difficult to imagine a real world policy-maker in such an enviable position, it would seem worthwhile to explore the implications of relaxing that assumption. The first two sections of this paper discuss the implications of uncertainty in the response to policy actions for the selection of optimal policy. Optimal policy in the presence of this type of uncertainty is found to differ significantly from optimal policy in a world of certainty. For example, in general all instruments are used, even if there is only one target variable. Analysis of the optimality question also provides some insight into what constitutes effectiveness.

In the third section it is shown that, not surprisingly, the way a structural change alters the effectiveness of policy depends on how it affects both the expected magnitude and the predictability of response to policy actions. The third section goes on to discuss briefly some of the problems involved in assessing the consequences of structural change for the effectiveness of policy when there are several instruments and several targets, and where the structural change affects the response of the system to disturbances as well as policy instruments.

I. *One Target—One Instrument*

It is instructive to discuss the complications uncertainty creates in a world of one target and one instrument before discussing the problem of optimal use of policy instruments when there are many instruments and targets. Suppose that the policy-maker is concerned with one target variable (y). Assume that y depends linearly on a policy instrument (P)—for example, government expenditures—and various exogenous variables—for example, autonomous investment demand. For our present purposes the impact of exogenous variables may be summarized in a single variable (u).

(1) $$y = aP + u$$

where a determines the response of y to policy action.

The policy-maker faces two kinds of uncertainty. First, at the time he must make a policy decision he is uncertain about the impact of the exogenous variables (u) which affect y. This may reflect his inability to forecast perfectly either the value of exogenous variables or the response of y to them. Second, the policy-maker is uncertain about the response of y to any given policy action. He may have an estimate $\bar{a}$ of the expected value of the response coefficient a in (1) above, but he is aware that the actual response of y to policy action may differ substantially from the expected value. At the time of the 1964 tax cut, for example, there was considerable uncertainty over the magnitude of the tax multiplier.

Both types of uncertainty imply that the policy-maker cannot guar-

antee that y will assume its target value (y^*). But they have quite different implications for policy action. The first type of uncertainty, if present by itself, has nothing to do with the actions of the policy-maker; it is "in the system" independent of any action he takes. The assumption that all of the uncertainties are of this type is one of the reasons Theil and others [1] [3] [4] [5] are able to prescribe "certainty equivalence" behavior; that is, that the policy-maker should act on the basis of expected values as if he were certain they would actually occur. Since in this case the variance and higher moments of the distribution of y do not depend on the policy action taken, the policy-maker's actions only shift the location of y's distribution.

In the presence of uncertainty about the response of y to policy actions, however, the shape as well as the location of the distribution of y depends on the policy action. In this case the policy-maker should take into account his influence on the variability of y.[2]

We will assume that the policy-maker chooses policy on the basis of "expected utility." In particular, we will follow Theil in assuming that the policy-maker maximizes the expected value of a quadratic utility function. In the one-target case this is simply:

$$(2) \qquad\qquad U = - (y - y^*)^2$$

where y^* is the target value of y.

The assumption of the quadratic enables us to restrict our attention to the mean and variance of y and to compare our findings directly with the familiar certainty equivalence results. The assumption of a quadratic is, of course, subject to the objection that it treats positive and negative deviations from target as equally important. The use of a fancier utility function would provide additional reasons for departing from certainty equivalence.

The precise relationship between policy actions and the variance of y is not obvious. In (1) above, for example, the policy-maker may believe that the response coefficient a is a random variable depending on some unobserved variables, and that it is correlated with u. In that case the y is a random variable with a variance given by:

$$(3) \qquad\qquad \sigma_y^2 = \sigma_a^2 P^2 + \sigma_u^2 + 2\rho\sigma_a\sigma_u P$$

where σ_u^2 and σ_a^2 are the variances of u and a, respectively, and ρ is the correlation coefficient between u and a.

On the other hand, it is possible to conceive of part of the uncertainty of y as the consequence of estimation error. Even if the policy-maker

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[2] Theil is of course aware that certainty equivalence behavior is not optimal in this case. In fact, he suggests that the sampling errors in the response coefficients are the most "dangerous" ones for a policy-maker who acts as if all random coefficient matrices coincide with their expectations [4, p. 74].

regards the "true" population response coefficient a as nonrandom, he may have to base his actions on an estimate of it obtained by fitting equation (1) to sample data. The estimate he uses will be a random variable, and its variance will affect the "variance" of y around its forecast value.[3]

As in the case where the population response coefficient is random, the magnitude of the policy action affects the contribution of this type of uncertainty to the variance of y. In this case, however, the contribution depends on the difference between the policy taken in the forecast period and the average level of policy pursued in the sample period used in estimating a. Assuming the u's are independent over time, a will be uncorrelated with the u for the forecast period and equation (3) may be rewritten:

$$(3') \qquad \sigma_y^2 = \sigma_a^2(P - \bar{P})^2 + \sigma_u^2$$

where $\bar{P}$ is the average P for the sample period on which the estimate of a is based. Although we will use the first formulation to illustrate the significance of uncertainty in the response of y to policy actions, our results can be translated easily for use in the forecast error case.

Assuming the response coefficient is a random variable, we may find the expected utility associated with a given policy action by substituting (1) in (2):

$$(4) \qquad E(U) = -\left[(\bar{y} - y^*)^2 + \sigma_y^2\right]$$

$$= -\left[(\bar{a}P + \bar{u} - y^*)^2 + \sigma_a^2 P^2 + \sigma_u^2 + 2\rho\sigma_a\sigma_u P\right]$$

where $\bar{y}$ and $\bar{u}$ are the expected values of y and u, respectively. There is no reason to suppose that $\bar{u}$ equals zero.

By differentiating (4) with respect to P and setting the derivative equal to zero, the optimal value of P is easily found to be:

$$(5) \qquad P^* = \frac{\bar{a}(y^* - \bar{u}) - \rho\sigma_a\sigma_u}{\bar{a}^2 + \sigma_a^2}$$

The optimal policy indicated by equation (5) clearly differs from the policy which would be pursued in a world of certainty or of certainty equivalence. The policy-maker should make use of more information than the expected value of the exogenous variables and of the response coefficient of a. Even when a and u are independently distributed, he

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[3] Hooper and Zellner [2] provide a discussion of the error of forecast for multivariate regression models. In general the variance of the forecast error is $\sigma_F^2 = \sigma^2(xX1^u)[1 + x_F'^{-1}x_F]$ where x is the matrix of sample observations on the independent variables, and x_F is the vector of deviations of the independent variables from their sample means for the forecast period.

should make use of information about the variance of a as well as its mean. If a and u are not independent, he also needs to know their correlation. The assumption of a quadratic utility function does not lead to certainty equivalence except when the policy-maker is certain about the effects of his actions. Another interesting implication of (5) is that it does not in general pay to aim directly at the target. If a and u are independent, for example, equation (5) can be rewritten to show that the fraction of the expected "gap" between $\bar{u}$ and y^* which should be filled by policy action depends only on the coefficient of variation of a:

$$(5') \qquad\qquad P^* = g/(1 + V^2)$$

where $g = (y^* - \bar{u})/\bar{a}$, the expected gap, and V equals $\sigma_a/\bar{a}$, the coefficient of variation of a.[4] Only if the policy-maker is absolutely certain about a $(V = 0)$ will he close the entire gap; so long as V is finite, he will partially fill the gap.

Some care must be used in interpreting this result. The gap in this context is not the difference between what policy was "last period" and what would be required to make the expected value of y equal to y^*. In the example we have used, the gap is the difference between P equal to zero—the point where the variance of y is least—and the P required to give an expected value of y equal to y^*. If the expected value of a and u and the standard deviation of a remained unchanged for several periods, the optimal policy would also remain unchanged—the policy-maker would not reduce the gap in successive periods. In the case of "forecasting error" the gap is the difference between the average value of policy in the sample used to estimate a and the P which gives $\bar{y}$ equal to y^*. In this case, if the parameter a were reestimated each period and the expected value of u remained the same, policy would be continually revised, making the expected value of y closer and closer to y^* in successive periods.

A natural question to ask is to what extent uncertainty about a affects optimal behavior. Equation (5) indicates that "moderate" uncertainty about a may have a substantial effect. Suppose, for illustrative purposes, that the monetary authority believes that the equation $Y = aM + u$ correctly specifies the relationship between the stock of money (M), an exogenous variable (u), and money income (Y). It puts its staff to work estimating the relationship and obtains an estimate of 5 for the value of a, significant at a "t" level of 2. Further suppose that for the sample of observations on Y and M used to estimate a, the average level of M was \$100. Now suppose that for the next period the desired level of Y is \$650 and the expected value of u is taken to be \$50. In a

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[4] (5') can be used for the estimation error case if P^* is interpreted as the deviation of optimal policy from the mean of P during the sample period, and u is defined to include $\bar{a}$ times that mean.

world of certainty M would simply be set at \$600/5 or \$120. Optimal policy in the uncertain situation confronting the policy-maker is most easily found by consulting equation (5'). If M were set at its average value during the sample period the expected value of Y would be \$550, leaving a gap of \$20 (\$100/5).

According to (5'), however, only 80 percent of this gap should be closed with a coefficient of variation of a equal to one-half. Hence it is optimal to set M at \$116 (=\$100+.8×\$20).

When a and u are not independent, the results are slightly more complicated. The policy-maker must now take into account the covariation
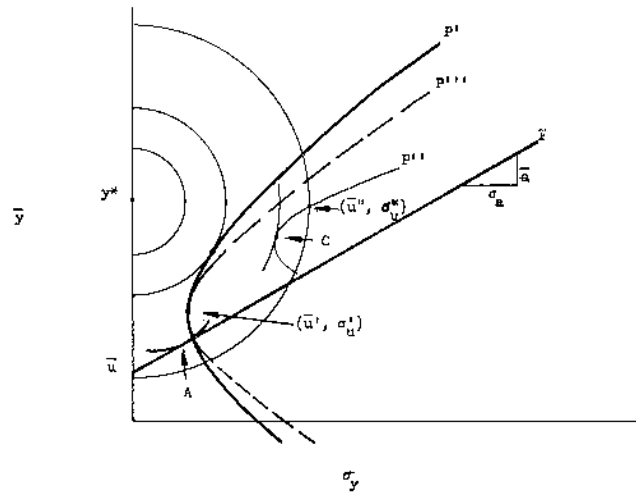

FIGURE 1

between the impact of his policy action and exogenous variables. If there is a positive correlation, it will pay him to shoot for a lower value of $\bar{y}$ than otherwise; if negative, higher. One interesting implication of this is that the fraction of the gap that the policy-maker should close will depend on which side of the target he is on. Perhaps more surprising, if a and u have sufficiently large positive correlation and there is a positive gap or sufficiently large negative correlation and there is a negative gap, it may actually pay for the policy-maker to go the "wrong" way! That is, at the initial point it may actually pay to reduce the variance of y at the expense of increasing the difference between expected y and y^*.

These results can be seen diagrammatically by consulting Figure 1 which shows the expected value of y on the vertical axis and the standard deviation of y on the horizontal axis. Indifference curves, showing various combinations of $\bar{y}$ and σ_y, which have the same expected utility, are drawn "around" y^*, the target value of y. These curves have the

form $(y^* - \bar{y})^2 + \sigma_y^2 =$ constant; i.e., they are concentric circles drawn around y^*.

In order to focus on the effects of uncertainty concerning the impact of policy, let us assume for the moment that σ_u is zero. Referring to Figure 1, suppose that in the absence of policy action, y is certain to be $\bar{u}$, which is well below y^*. By increasing P by "1 unit," the policy-maker may close the expected gap between y and $\bar{y}$ by an amount $\bar{a}$, but in so doing he also increases the standard deviation of y by σ_a.[5] The line P shows the possibilities open to the policy-maker. Optimal policy corresponds to point A. Because the indifference curves are horizontal where they leave $\bar{y}$ axis, and vertical at a value of $\bar{y} = y^*$, while the slope of the line P is $\bar{a}/\sigma_a$, it always pays to do something, but it never pays to aim for y^*. It is also apparent that reductions in σ_a for a given $\bar{a}$ increase the optimal amount of policy.

These results are not altered for σ_u not equal to zero, so long as the correlation between u and a is 0. In Figure 1 the point $(\bar{u}', \sigma_{u'})$ indicates the expected value and variance of y in the absence of policy action and the line drawn through that point labeled P' indicates the opportunities available to the policy-maker. This "opportunity locus" is curved, reflecting the absence of perfect correlation between a and u. This independence guarantees that unless $\bar{u} = y^*$ it will be optimal to pursue some policy action, for the opportunity locus is vertical at the point $(\bar{u}', \sigma_{u'})$ indicating that the first little bit of policy can be undertaken without increasing the variance of y.[6]

In Figure 1 the line P'' shows the way correlation between a and u alters the opportunity locus available to the policy-maker. In the example shown, a zero level of policy would leave the economy at the point $(\bar{u}'', \sigma_{u''})$. The positive correlation between a and u tilts the locus clockwise through that point so that a small decrease in P will decrease the variance of Y. Optimal policy involves choosing point C, which in this case involves going the "wrong" way. If by chance $\bar{a}(y^* - \bar{u}) = \rho\sigma_u\sigma_a$, the policy-maker should do nothing.

Figure 2 shows the way in which the appropriate level of policy action depends on the size of the gap. For given values of $\bar{a}, \rho, \sigma_a, \sigma_u$, the larger the gap, the more the policy-maker should do. With independence of a and u this relationship is linear-homogeneous. When a and u are correlated the relationship is linear, but a zero level of policy is optimal for some nonzero gap.
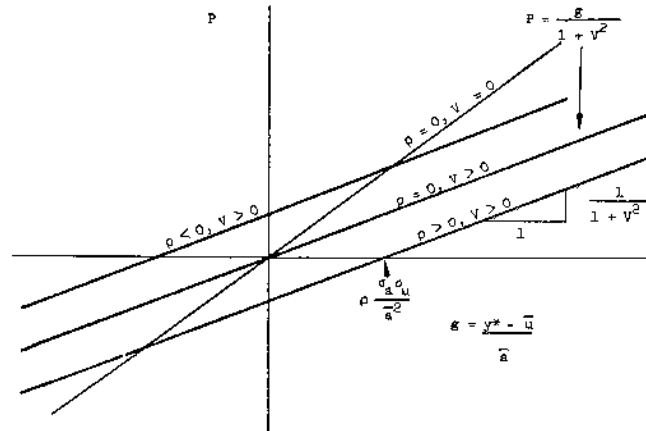
FIGURE 2

II. *Multiple Instruments*

One of the lessons of the theory of policy under certainty is that, in general, the attainment of n targets requires n instruments. If more than n instruments are available, the policy-maker is free to choose n arbitrarily in order to achieve his objectives. It should already be obvious that this rule breaks down under uncertainty. We will first show that with one target and two instruments it will generally be optimal to use some combination of both instruments.[7] It can similarly be shown that in general all instruments available should be used in pursuing one target. It follows that the addition of an objective requires some sacrifice in performance vis-à-vis objectives already being considered.

Suppose that:

$$(6) \qquad y = a_1 P_1 + a_2 P_2 + u$$

where again it is desired to maximize the expected value of the utility function given in (2). It will be convenient to define units of policy such that the $\bar{a}_i$ associated with each policy instrument is exactly 1. For simplicity let us assume that correlation between each a_i and u is 0. Then by differentiating the expected value of (2) with respect to P_1 and P_2 we find the conditions for optimal policy to be:

$$(7a) \qquad 0 = (P_1 + P_2 + \bar{u} - y^*) + P_1 \sigma_{a_1}^2 + \rho_{12} \sigma_{a_1} \sigma_{a_2} P_2$$

$$(7b) \qquad 0 = (P_1 + P_2 + \bar{u} - y^*) + P_2 \sigma_{a_2}^2 + \rho_{12} \sigma_{a_1} \sigma_{a_2} P_1$$

where ρ_{12} is the correlation between a_1 and a_2.

[7] The idea of looking at the problem of optimal policy behavior in an uncertain world as essentially a problem of portfolio choice arose in discussions with Arthur Okun around 1962.

Solving for the ratio of P_1 to total policy impact (P_1+P_2) we obtain:

$$(8) \qquad P_1/(P_1 + P_2) = \frac{\sigma_{a_2}^2 - \rho_{12}\sigma_{a_1}\sigma_{a_2}}{(\sigma_a{}^2 - 2\rho_{12}\sigma_{a_1}\sigma_{a_2} + \sigma_{a_1}^2)}{}^8$$

Equation (8) indicates the proportions in which the policy-maker should use the two policy instruments. The optimal policy portfolio as shown in (8) combines the instruments so as to minimize the coefficient of variation of their combined impact. Under the assumption of independence between the a_i's and the u, this policy portfolio can be treated as a single instrument and its optimal level determined as in Section I.
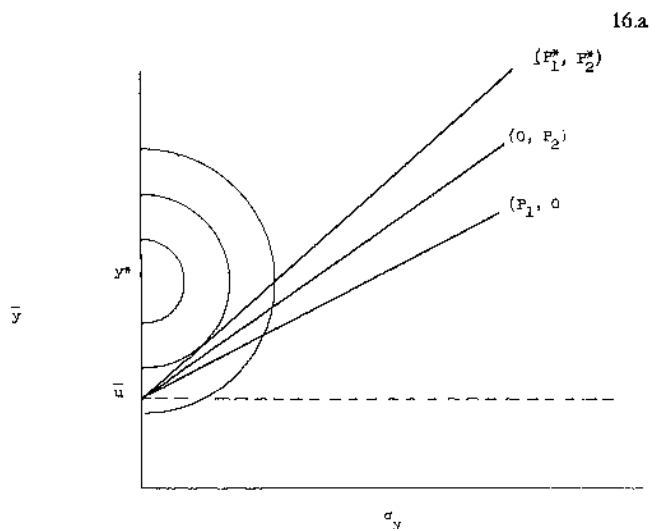
FIGURE 3

The coefficient of variation of the optimal policy package is, of course, less than or equal to the coefficient of variation of any single instrument. Figure 3 shows, for the special case where σ_u and ρ equal zero and where the ratio of the coefficient of variation of a_2 to that of a_1 is .8, the locus for the optimal combination of two policies compared to the loci available for each of the instruments used separately. Figure 3 assumes that the correlation between the impacts of the two instruments is zero. The presence of correlation between the a's complicates the computation of the optimal policy portfolio but does not alter the basic conclusion that several instruments are better than one in the pursuit of one goal.

The optimal amount of P_1 per "unit" of combined policy action

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[8] By virtue of the normalization of a_i, of course, the standard deviations are coefficients of variation.

$$\frac{P_1}{P_1+P_2} = \frac{\sigma_{a_2}}{\sigma_{a_1}+\sigma_{a_2}}$$

$$\frac{P_1}{P_1+P_2} = \frac{\sigma_{a_2}^2 - \rho\sigma_{a_1}\sigma_{a_2}}{\sigma_{a_2}^2 - 2\rho\sigma_{a_1}\sigma_{a_2}+\sigma_{a_1}^2}$$

$$\rho = \frac{\sigma_{a_2}}{\sigma_{a_1}}$$

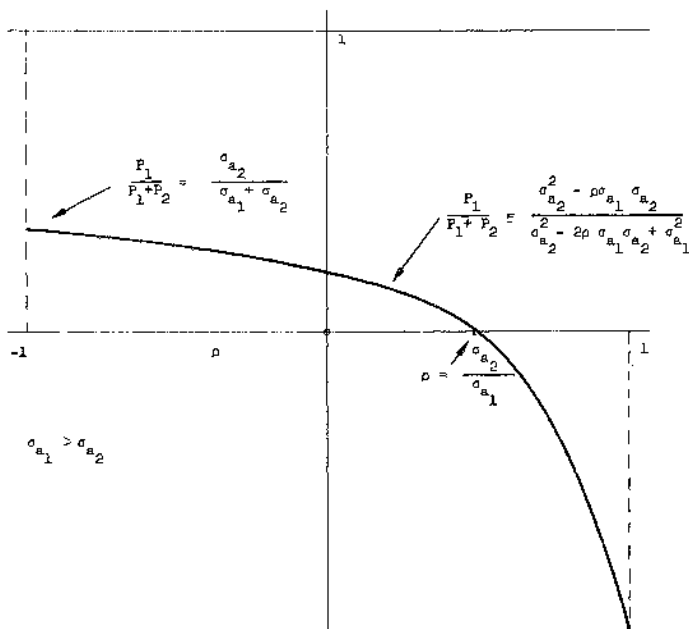$$\sigma_{a_1} > \sigma_{a_2}$$

FIGURE 4

P_1+P_2 is shown in Figure 4 as a function of the correlation between a_1 and a_2. In the example shown it is assumed that the coefficient of variation of a_1 is greater than that of a_2. From Figure 4 it can be seen that for sufficiently high positive correlation it is optimal to use P_1 (the "less efficient" instrument) in the "wrong" direction.

Table 1 shows the coefficients of variation for the optimal policy

TABLE 1

COEFFICIENT OF VARIATION FOR ρ_{12} OPTIMAL POLICY PACKAGE

		−9/10	−3/4	−1/2	−1/4	0	1/4	1/2	3/4	9/10	1
	1.0	.22	.35	.50	.61	.71	.79	.87	.94	.97	1.0
	1.1	.23	.37	.52	.64	.74	.83	.90	.97	1.00	0
	1.2	.24	.39	.54	.67	.77	.86	.93	.99	.99	0
	1.6	.27	.43	.61	.74	.85	.93	.99	.98	.85	0
σ_{a_1}	2.0	.30	.47	.65	.79	.89	.97	1.00	.94	.74	0
—	2.4	.31	.49	.69	.82	.92	.99	1.00	.89	.67	0
σ_{a_2}	2.8	.33	.51	.71	.85	.94	.99	.99	.86	.63	0
	3.2	.34	.53	.73	.86	.95	1.00	.98	.83	.60	0
	4.0	.35	.55	.76	.89	.97	1.00	.96	.80	.56	0
	8.0	.39	.60	.81	.93	.99	.99	.92	.73	.49	0

$\sigma_{a_2} = 1$.

package for various values of ρ_{12}, σ_{a_1}, σ_{a_2}. The table is constructed with the variance of a_2 normalized at one and with $\sigma a_2 < \sigma a_1$. Hence the lowest coefficient of variation of policy impact that would be obtained by using a single policy is one. The improvement realized by using both instruments rather than P_2 alone is indicated by the difference between one and the appropriate entry.

The gain from diversification of policy instruments is not a simple function of the correlation coefficient; it also depends on the ratio of coefficients of variation of the two policy instruments. As can be seen from Table 1, negative correlation between a_1 and a_2 greatly assists in reducing the variance in the impact of the policy package, the reduction being greatest for cases where the variances of the two instruments are equal. As the correlation increases, the gain from using two instruments decreases, until at some level of positive correlation none of policy P_1 is used. For correlation greater than that amount, P_1 will be used in the "wrong" direction and some reduction in variance will be realized. In the extreme cases of perfect positive or perfect negative correlation between the a's there exists a policy package with zero variance.

If there is correlation between the disturbance and the impact of either of the policy instruments, it should also be taken into account in the selection of a policy package. Other things being equal, increasing the correlation between the impact of a particular instrument and the disturbance will decrease its use relative to other instruments. The importance of such correlation depends on the size of the gap: the larger the gap, the less the relative contribution of the disturbance to the variance of y after policy action and the less important the correlation of the disturbance and instruments.

The optimal use of n instruments follows the principles illustrated with two; if the error term is independent of the policy response coefficients, the portfolio of instruments which has the lowest coefficient of variation should be chosen. In general this will involve using all of the instruments, and it may involve using some instruments the "wrong" way.

Generalization to many targets is conceptually simple but algebraically tedious. Solution of a multiple-goal problem requires specification of a multidimensional utility function which, if it is quadratic, implicitly provides weights for trading off expected values, variances, and covariances of the policy objectives. The particular solution obviously depends on the weights imbedded in the utility function. One feature of the results is perhaps worth noting: since all policy instruments would be used in pursuit of a single target, improvement in performance vis-à-vis one objective requires sacrificing other objectives—even when the number of instruments exceeds the number of objectives.

III. *Structural Change and the Effectiveness of Policy*

The evaluation of the impact of any particular structural change on the effectiveness of policy is extremely difficult.[9] For our purposes a structural change is described by the way it changes the joint distribution of the parameters a_i and u. The task of determining how the imposition of some new regulation or the emergence of some new financial market alters that distribution is obviously a major one and beyond the scope of this paper. Our relatively modest objective here is to indicate the empirical questions which the above analysis suggests are of importance.

A Single Instrument. First let us consider structural changes which affect only the impact of policy actions; i.e., which do not affect either the expected value or dispersion of the disturbance term. In addition, let us assume that the structural change in question does not alter the correlations among the policy impact coefficients and the disturbance. The effect of such a change on expected utility can be found directly by substituting the value of optimal policy (expressed in terms of $\bar{a}$, σ_a, ρ, $\bar{u}$, etc.) in the equation for expected utility (4). Alternatively, the consequences of structural change can be seen by noting how the "opportunity locus" in Figures 1 and 3 are affected. By substituting $(\bar{y} - \bar{u})/\bar{a}$ for P in equation (3) we find:

$$(9) \qquad \sigma_y = \left(\frac{\sigma_a^2}{\bar{a}^2} (\bar{y} - \bar{u})^2 + 2\rho \frac{\sigma_a}{\bar{a}} \sigma_u (\bar{y} - \bar{u}) + \sigma_u^2 \right)^{1/2}$$

It is immediately clear that a structural change which does not affect the coefficient of variation of a (nor ρ, $\bar{u}$, and σ_u) does not alter the opportunity locus and hence does not alter the expected utility derived from optimal policy. It is also clear that increasing the coefficient of variation, for ρ equal zero, results in an opportunity locus which has a larger variance for every value of $\bar{y}$ (except for zero policy). Hence increasing the coefficient of variation leads to a reduction in the effectiveness of policy. Such a shift is illustrated in Figure 1 by a movement of the opportunity locus labeled P' to the location indicated by the dotted locus labeled P'''.

One of the simplest illustrations of this result is the argument that increasing the legal reserve requirement, thereby reducing the expected response to a given sized change in reserve base, actually increases the effectiveness of monetary control. Suppose one believes that the money

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[9] What is classified as a structural change obviously depends on the specific problem under discussion. The removal of the ceiling rate on time deposits, for example, would change the response of financial markets to open market operations; that is, it would change the structure within which the Federal Reserve conducts day-to-day policy. At the same time, however, the ceiling rate itself could be used as a policy instrument to influence target variables. While some changes in the "structure" are the direct consequence of actions by the monetary authority, others may be exogenous from their point of view.

stock is all that matters and, further, that the money stock (M) is related to the reserve base (R) by the following equation:

$$(10) \qquad M = \frac{1}{k + \epsilon} R$$

where k is the legal reserve requirement and ϵ is banks' demand for free reserves (expressed as a fraction of their deposit liabilities). Suppose further that, from the viewpoint of the monetary authority, ϵ is a random variable with a known distribution. Then it is easy to see that, for reasonable distributions of ϵ, increases in k will reduce the expected response of M to R but can reduce the standard deviation of that response more than proportionately, thereby reducing its coefficient of variation. One of the appeals of the 100 percent reserve proposal, of course, is the fact that as k approaches one, the coefficient of variation of the response of M to R goes to zero.

If the correlation between the impact of the policy instrument and u is not zero, matters are slightly more complicated. From (9) it is apparent that increases in σ_a shift the opportunity locus to the right for some values of $(\bar{y} - \bar{u})$ and to the left for others. This is illustrated for a positive ρ in Figure 5 below where the dotted locus corresponds to a higher σ_a. As indicated in Figure 5, the minimum risk that can be obtained is independent of the value of σ_a, and depends only on the value of σ_u and ρ. Hence, it is possible to get the paradoxical result that an increase in the dispersion of response can make policy more "effective" for some set
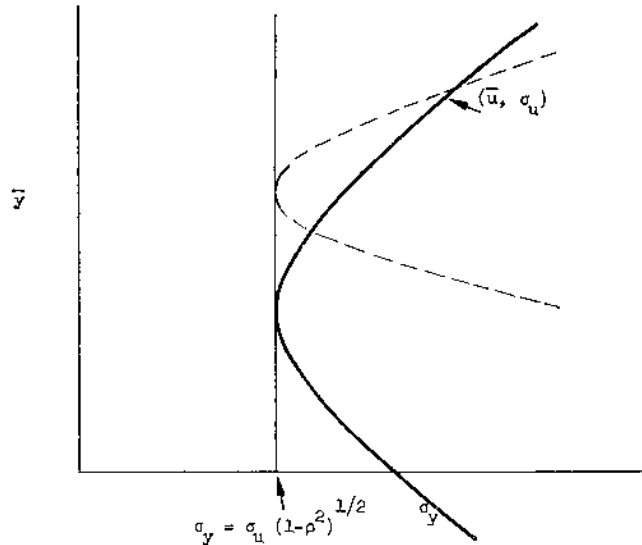
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$$\sigma_y = \sigma_u (1-\rho^2)^{1/2}$$

FIGURE 5

of values of $\bar{u}$ and σ_u. This peculiar result is a consequence of the fact that it is possible to reduce the variance of y below σ_u whenever μ and a are correlated. Consider, for example, the case where a and u are positively correlated, and where $\bar{u}$ is less than y^*. In this case a reduction in risk can be obtained for "negative" values of P; i.e., by setting $\bar{y}$ even further from y^* than $\bar{u}$. Suppose that the optimal policy initially involves going in the counterintuitive direction in order to take advantage of such risk reduction. With a larger σ_a the same reduction in risk can be obtained without pushing $\bar{y}$ as far away from y^*; hence it will result in a higher expected utility.

Multiple Instruments. The consequence of structural change when there are a number of instruments is closely analogous to the single instrument case. If the structural change does not affect the coefficient of variation for the individual instruments, their correlations, or the distribution of the disturbance, the opportunity locus is unaltered. If the coefficient of variation of an individual instrument is increased, the consequences can be analyzed in two steps.[10] First, the effect of the change on the coefficient of variation of the optimal policy package can be found. Once this is determined, its consequences for the performance of the policy-maker can be analyzed exactly as in the single instrument case.

The consequences of increasing the coefficient of variation of one instrument are illustrated in Table 1.[11] Two points illustrated in Table 1 are worthy of comment. First, so long as both instruments are used in the same direction (see Figure 4), increasing the coefficient of variation of one instrument increases the coefficient of variation of the optimal policy package. If an instrument is used in the counterintuitive direction, however, increasing its coefficient of variation improves the optimal policy package. Second, the table illustrates the obvious fact that decreasing the effectiveness of one instrument which is being used together with others has much less effect than if the instrument were being used by itself. For example, even with a correlation of $-9/10$, the consequence of increasing σ_{a_1} from 1 to 2 is to increase the coefficient of variation of the optimal policy package by less than 40 percent.

Multiple Targets. The presence of a number of target variables in the utility function greatly complicates the task of evaluating the consequences of a given structural change. It is quite unlikely that a change will affect the opportunity locus for different targets in the same way. Hence the desirability of the change will depend on the relative weights placed on the different targets in the utility function. For example, it is commonly argued that the development of the Euro-dollar market and

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[10] Assuming the correlations between the a_i's and u are zero.

[11] Table 1 shows the effect of increasing the coefficient of variation for the less effective of the two instruments.

the general increase in international capital mobility in recent years has made monetary policy a more effective tool in stabilizing the balance of payments, while at the same time reducing its usefulness for controlling domestic economic disturbances.

Disturbances. The discussion thus far has focused on the consequences of structural change for the performance of the policy-maker when confronted with a given distribution of disturbances. A structural change which alters the response to policy actions is also likely to alter the magnitude of the problems with which he must cope. In the above example, greater capital mobility presumably increases the responsiveness of the U. S. balance of payments to disturbances; e.g., in this case to changes in the interest rates in other countries. Thus a structural change which gives the U. S. a more effective tool may also have given it a more difficult task to perform. If this is the case it is not clear that such a structural change is desirable. This can be illustrated by reference to the single instrument case with ρ equal to zero. Suppose for the moment that a structural change doubles $\bar{a}$ and σ_a, but also doubles σ_u, without altering $\bar{u}$. From Figure 1 it is clear that such a change leaves the policy-maker with a worse opportunity locus.[12] If, in addition, there is an increase in the variation of $\bar{u}$ from period to period, still greater losses result.

In the real world, evaluating the desirability of a given structural change is further complicated by the fact that a change which decreases the response of the system to one type of disturbance is likely to increase its response to another. Consider, for example, the consequences of a permanent fixed ceiling rate on time deposits. It can be argued that such a restriction on banks' competition for deposits reduces the response of the system to shifts in the demand of bank borrowers for loans. At the same time, the existence of such a ceiling may increase the destabilizing effect of shifts in depositors' preferences between the liabilities of banks and other financial intermediaries. In general, then, evaluation of some particular structural change requires an empirical judgment as to the relative importance of various kinds of disturbances.

[12] Again, when $\rho \neq 0$, it is possible to have the paradoxical result that increasing σ_u is advantageous.

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