THE TIMING OF MARKETS AND MONETARY TRANSFERS IN CASH-IN-ADVANCE ECONOMIES

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In cash-in-advance models, do the timing of markets and the timing of the monetary transfer affect equilibrium money demand? The timing of markets generates different individual money demands; however, under the common assumption that agents are identical, these differences do not affect the behavior of equilibrium real balances. In contrast, the timing of the monetary transfer has important implications for agent's information sets; these implications can influence the equilibrium characteristics of real balances.

I. INTRODUCTION

The search for a general equilibrium model in which the demand for money is explicitly derived from individual optimization has led many theorists to adopt a cash-in-advance constraint. By requiring that a subset of expenditures be financed out of current money holdings, the cash-in-advance constraint captures the transactions motive of money demand and, under fairly general conditions, leads to a finite nominal price level in equilibrium. The attractiveness of finance constraint models in comparison with alternative general equilibrium monetary models, specifically overlapping generations models and models that place money in the direct utility function, stems from two features: First, unlike models in which money demand characteristics result from an arbitrarily chosen utility function, the use of the cash-in-advance constraint allows an explicit characterization of the traditional components of money demand. Second, in overlapping generations models, money serves only as a store of value so that, in the absence of additional assumptions, the introduction of alternative assets often leads to non-monetary equilibria. This is not the case with finance constraint models due to the severe nature of the constraint.

Cash-in-advance models are inherently sequential, thus necessitating an explicit modeling of the timing of transactions and the flow of money within the economy. For instance, the manner in which the money stock is carried over from one period to the next, i.e., whether in the form of households' nominal balances or firms' sales revenues, must be addressed with the choice implying possible different equilibrium characteristics. (For a discussion of this issue, see MacKinnon [1987].) In general, the sequential evolution of cash-in-advance economies implies that exogenously specified timing choices will influence individual demands for both goods and assets and equilibrium behavior. This paper attempts to clarify the implications of two such timing conventions: (i) the timing of markets and (ii) the timing of the monetary injection.

The potential importance of market timing was initially highlighted in papers

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by Robert E. Lucas, Jr. [1982] and Lars E. O. Svensson [1985b]. Specifically, in an analysis of exchange rates Lucas applied the cash-in-advance constraint to the acquisition of consumption goods, thereby implying the existence of two distinct markets: one for goods and another for assets. Lucas assumed that the state of the world was revealed at the beginning of the period and arranged the markets so that the asset market preceded the goods market. With the opportunity to revise nominal money holdings after the state of the world was known, individuals would never hold more money than needed for transactions if nominal interest rates were positive. Lucas made the additional assumption that nominal interest rates were indeed positive and demonstrated that, under this restriction, unit-velocity quantity theoretic results were obtained. Svensson [1985b] modified Lucas' model by reversing the order of markets. That is, Svensson placed the goods market first so that agents did not have the opportunity to revise their money holdings after learning of the state of the world. As Svensson states:

> My model differs from Lucas's in that consumers must decide on their cash belongings before they know the current state and hence before they know their consumption. This gives rise to a combined transactions, precautionary, and store-of-value demand for money [1985b, 921].

Svensson later reports, however, that this seemingly important change in individual money demand had absolutely no consequences for the equilibrium characteristics of real balances. Rather,

> ...the behavior of real balances and, in particular, whether or not the liquidity constraint is binding turns out to be independent of whether the asset market opens before or after the state is known [1985b, 938].

In other words, the inclusion of a precautionary motive in individual money demand had no impact on the behavior of equilibrium, aggregate money demand.

This difference between the implications of market timing for individual and aggregate money demand has not been appreciated in the cash-in-advance literature and continues to be a source of confusion in current research as evidenced by the following statements.

In an empirical analysis of cash-in-advance (CIA) models, Hodrick, Kocherlakota, and Lucas [1991] motivate their use of Svensson's timing rather than Lucas's (referred to as "simple CIA models") in the following manner:

> Simple CIA models require that the amount of money held in a given period be at least sufficient to cover perfectly anticipated expenditures. Agents faced with positive nominal interest rates never hold idle cash balances in these economies, so the entire money supply turns over each period. Consequently, these models make the incorrect prediction that the consumption velocity of money is always unity.
>
> In response to this difficulty, Lucas [1984] and Svensson [1985b] modify the information structure in the basic CIA setup....Cash balances must be chosen before the quantity of output is known. Therefore, agents may choose to carry unspent cash across periods, and velocity can in principle vary [1991, 359].

As an additional example, Thomas Sargent and Bruce Smith [1988] state:

> It is well known that the monetary and real aspects of these models hinge on the assumed timing of transactions. There are several papers that study consequences of varying such assumptions. For instance Svensson [1985b], Lucas [1984], and Lucas and Stokey [1987] consider the impact of varying the timing of transactions relative to when various elements at the state of nature are revealed to agents. Such a variation permits a "precautionary demand" for money to emerge (emphasis added).

And, in a survey of recent developments in macroeconomics, Stanley Fischer [1988] writes:

> Svensson [1985b] relaxes the unitary velocity assumption by assuming that individuals face uncertainty about their spending and have to decide on money holdings before the uncertainty is resolved [1988, 300].

Since all quotes are in the context of discussing the equilibrium, aggregate characteristics of the models, these statements are in direct contradiction to Svensson's.

In order to sort out these issues, section II examines in detail the individual's maximization problems implied by the two market structures. While paralleling the earlier works of Svensson and Lucas, this detailed derivation facilitates the comparison of individual money demand in the two models which, in turn, permits the identification and characterization of the transactions, precautionary, and store-of-value motives of money demand. Due to the assumption of identical agents employed by both Lucas and Svensson (as well as the papers quoted above), the differences in individual money demand are eliminated in the construction of equilibrium real balances. The reason for the equivalence of the two models is straightforward. While the Lucas timing permits revision of money holdings upon learning of the state of the world, the assumption of identical agents implies equilibrium asset prices will be sustaining prices; i.e., asset trade is zero. Consequently, the two models generate the same separating hyperplane that defines equilibrium real balances.

Section III discusses the implications of the second timing choice, i.e., the receipt of the monetary transfer. Both Lucas [1982] and Svensson [1985b] introduce new money as a lump-sum transfer (or tax if the monetary growth rate is negative). Additionally, both authors assume that the transfer is received in the asset market; hence, at the beginning of the period in Lucas's model and at the end of the period in Svensson's. Somewhat surprisingly (since the transfer is lump-sum), this timing choice can influence equilibrium real balances. The reason is the implied difference in agents' information of the aggregate money stock. Specifically, the end-of-period transfer gives agents perfect information about the money stock available in the goods market in periods t *and* t+1. As a consequence, the end-of-period transfer seems an inappropriate modeling choice for much of the analysis that employs the cash-in-advance framework. That is, the cash-in-advance model is a useful paradigm for the analysis of the implications of monetary uncertainty as, for instance, captured in the serial correlation and conditional variance of the monetary growth rate. It is precisely these types of questions, however, that can not be addressed if the monetary transfer is received at the end of the period since, for all practical purposes, monetary uncertainty is eliminated.

II. THE INDIVIDUAL'S PROBLEM

This section derives those conditions that characterize an individual's optimum in models that are identical except for the timing of markets. Using terminology introduced by Kohn [1984], I refer to the sequence of markets in which the asset market precedes the goods market as the liquid asset model and the reverse order as the semi-liquid asset model.[1] In what follows, uncertainty is due to randomness in the growth rate of the money supply (g_t) and the level of the endowment (x_t) (i.e.,

1. This terminology is explained as follows: If the goods market follows the asset market, all assets can be converted into currency to finance that period's consumption. If the goods market precedes the asset market, only beginning-of-period currency can finance consumption that period; the asset market, however, is a barter market. The "illiquid asset" model imposes the cash-in-advance constraint on goods and asset purchases.

both models are exchange economies). The support of g is the interval $G = (-1, \infty)$ while that of x is the interval denoted $X = (0, \infty)$. Let $s_t = (g_t, x_t)$ denote the state of the world at time t. It is assumed that s_t follows a stationary Markov process with the transition function given by $H(s,s') = \Pr(g_{t+1} < g',\ x_{t+1} < x' \mid g_t = g,\ x_t = x)$ where $s \in S = GxX$. Agents know s_t at the beginning of period t.

Individuals are assumed to maximize

$$\sum_{t=0}^{\infty} \beta^t \mathrm{E}[U(c_t)] \tag{1}$$

with E the expectations operator, β the subjective discount factor ($0 < \beta < 1$), and U assumed to be strictly concave, continuously differentiable, and bounded. Two securities exist in the economy: money (M) and a mutual fund (z) that entitle the owner to a fraction of the nominal value of the endowment process. It is assumed there is one outstanding share of the mutual fund.

Analysis of equilibrium is restricted to the class of stationary monetary equilibria in which the equilibrium quantity of real money balances and the price of the mutual fund measured in units of the consumption good are functions of the state but not the date. It is assumed that agents know these functions along with the transition function $H(s,s')$; their expectations are rational.

The Liquid Asset Approach

Following Lucas [1982], the household consists of two individuals—a buyer and a seller. At the beginning of the period, the state of the world is revealed (s_t) and, consequently, the household determines its portfolio and consumption plans. The buyer first visits the securities market where optimal quantities of money and shares of the security are purchased with the household's wealth. This wealth is in the form of money and shares of the security carried over from the previous period plus the new monetary transfer. The buyer then visits the goods market where consumption is financed out of current money holdings. Meanwhile, the seller receives the dividends from current security holdings; these are sold to a buyer from another household for cash which must be carried over to the next period. This scenario implies the following nominal budget constraints:

$$W_t = M_t + Q_t z_t \tag{2}$$

$$M_t \geq P_t c_t \tag{3}$$

$$W_{t+1} = (M_t - P_t c_t) + z_t(P_t x_t + Q_{t+1}) + g_{t+1}\overline{M}_t \tag{4}$$

W_t is nominal wealth, Q_t is the dollar price of a share of the mutual fund, $\overline{M}_t$ is the money stock, and P_t is the dollar price of a unit of consumption. Using c as numeraire allows (2), (3), and (4) to be expressed as

$$w_t = m_t + q_t z_t \tag{5}$$

$$m_t \geq c_t \tag{6}$$

$$w_{t+1} = (m_t - c_t)/\alpha_{t+1} + z_t(x_t/\alpha_{t+1} + q_{t+1}) + g_{t+1}\overline{M}_t/P_{t+1} \tag{7}$$

where $\alpha_{t+1} = P_{t+1}/P_t$ defines the one-period inflation rate. As noted, equilibrium real prices and real balances (denoted by lower case variables) will be functions of the current state only. Consequently, time subscripts will be suppressed and primes used to denote values at time t+1.

The individual's utility maximization problem can be expressed in the form of a dynamic programming problem in which the value function is given by[2]

$$(8)\quad v(w,s) = \max\{U(c) + \beta \int v(w',s')dH(s,s')\}$$

subject to (5) - (7).

Notice that money balances are not a direct argument of the value function since agents have the opportunity to adjust their money holdings in the securities market before making their consumption purchases. Let Γ denote the Lagrange multiplier associated with real wealth and μ the multiplier associated with the finance constraint. Then maximizing (8) subject to (5) and (6) with w' given by (7) and assuming $c, z > 0$ yields the following Kuhn-Tucker conditions:

$$(9)\quad U_c(c) = \mu + \beta \int (\Gamma'/\alpha')dH(s,s')$$

$$(10)\quad -\Gamma + \mu + \beta \int (\Gamma'/\alpha')dH(s,s') = 0$$

$$(11)\quad \Gamma q = \beta \int \Gamma'(x/\alpha' + q')dH(s,s')$$

2. The proof that $v(w,s)$ exists and is strictly concave and differentiable is a direct application of Lucas' [1978] proof. By introducing the slack variable $b(s) = m(s) - c(s)$ it is easy to see the connection between this model and his. With this change, the dynamic programming problem becomes

$$v(b,z,s) = \max\{U(c) + \beta \int v(b',z',s')\, dH(s,s')\}$$

subject to

$$c_t + b_t + q_t z_t < b_{t-1}\alpha_t + z_{t-1}(x_{t-1}/\alpha_t + q_t) + g\overline{M}_{t-1}$$

$$c_t \geq 0;\ b_t \geq 0;\ z_t \geq 0$$

The individual, given his wealth, chooses optimal consumption, end-of-period real balances and end-of-period shares. The individual's problem, therefore, is identical to that in Lucas [1978] so that all proofs presented there apply. For clarity of exposition I will use the statement of the individual's problem as expressed in (5) - (6).

and the complementary slackness conditions: $\mu[m - c] = 0$; $\mu \geq 0$ and eq. (6).

Using (9) and (10) yields

$$(12)\quad U_c(c) = \Gamma$$

$$(13)\quad \mu = U_c(c) - \beta \int [U_c(c')/\alpha']dH(s,s') \geq 0$$

The interpretation of these conditions highlights the characteristics of the liquid asset model. Equation (12) implies the marginal utility of consumption always equals the marginal utility of real wealth. This is due to the symmetry that real balances and other forms of wealth have in this model as a result of the securities market preceding the goods market. In other words, the fact that agents have the ability to adjust their money holdings after the state of the world is revealed and before they face the finance constraint implies that agents choose their portfolio without regard to the liquidity characteristics of an asset since any form of wealth can be converted into consumption. Equation (11) is a standard capital asset pricing equation for z. As is usual, for the individual to be at a maximum the utility foregone from the purchase of a unit of z must be equal to the discounted expected utility that ownership will provide next period. Since this paper will concentrate exclusively on the determination of equilibrium real balances, no further analysis of eq. (11) will be carried out. To interpret equation (13), note it may be rewritten as

$$(14)\quad U_c(c)/P_t = \mu/P_t + \beta \int [U_c(c')/P_{t+1}]dH(s,s')$$

The left-hand side represents the loss in utility from the acquisition of another dollar. At the margin, this amount must be equal to the utility gain that a dollar represents. This is given by the right-hand side of eq. (14): μ/P_t is the utility value of the liquidity services of a dollar (i.e., the "dividend") while the second term represents the discounted expected utility increases (or decreases) due to capital gains (or losses) resulting from price level changes.

The Semi-Liquid Asset Approach

This section presents a model similar to Svensson's [1985b] paper.[3] Now, agents must visit the goods market first; there is no asset market to "fine tune" nominal balances after the state is revealed. Monetary transfers are received at the beginning of the period and dividends from last period's security holdings are received while the buyer is en route to the securities market. This timing implies the following nominal budget constraints:

(15) $$W_t = M_t + Q_t z_t + P_t c_t$$

(16) $$M_{t-1} + g_t \overline{M}_{t-1} \geq P_t c_t$$

(17) $$W_{t+1} = M_t + z_t(P_{t+1}x_{t+1} + Q_{t+1}) + g_{t+1}\overline{M}_t$$

Changing the order of markets results in two differences between the semi-liquid and liquid models; these are reflected in eqs. (16) and (17). Most importantly, the money that is used to finance current consumption was chosen in period $t-1$ as shown in eq. (16).[4] Also, since cash from the sale of dividends is received en route to the security market, dividends are due to shares purchased in the previous period. This change can be seen by comparing the first term in the parentheses in eq. (17) with that in eq. (7). Using c as numeraire yields

(18) $$w_t = M_t/P_t + q_t z_t + c_t$$

(19) $$m_t = (M_{t-1} + g_t \overline{M}_{t-1})/P_t \geq c_t$$

(20) $$w_{t+1} = (M_t/P_{t+1}) + z_t(x_{t+1} + q_{t+1}) + g_{t+1}\overline{M}_t/P_{t+1}$$

Again, the individual's problem can be expressed in the form of a dynamic programming problem with the value function given by

(21) $$v(w,s,M_{t-1}) = \max[U(c) + \beta \int v(w',s',M_t)dH(s,s')]$$

subject to (18) – (20).[5]

Notice that M_{t-1} directly enters the value function in contrast to the previous section. That is, since both money and real wealth can constrain consumption, these are arguments in the value function. More formally, w and M_{t-1} are both individual state variables in the semi-liquid model while only wealth is in the liquid asset model.

3. To show that the timing of markets has no substantive effects on equilibrium, it is necessary to have the rest of the economic environment identical. Therefore, I have adopted Lucas's convention for the timing of the monetary transfers, i.e., the transfer is received after the state is revealed and before markets open. In contrast, Svensson had individuals receiving the monetary transfer at the end of the period. The implications of the timing of the monetary transfer are discussed in section III.

4. As mentioned in footnote 3, the money that is used to finance consumption includes both money chosen in the last period and the lump-sum monetary transfer.

5. Again, proofs of the existence of v and its properties are due to Lucas [1980]. That v exists is easy to establish by standard contraction mapping arguments. The differentiability of v is proven by noting that in the two regions $m > c$ and $m = c$ (letting $m = (M_{t-1} + g_t\overline{M}_{t-1})/P_t$) the one-sided derivatives of v on the boundary are the same. (See Lucas [1980] prop. 3 for details.)

As in the previous section, time subscripts will be suppressed while Γ and μ denote the Lagrange multipliers associated with real wealth and real balances respectively. Then maximizing eq. (21) subject to eqs. (18) and (19) with w' given by eq. (20) yields the following Kuhn-Tucker conditions:

$$U_c(c) = \Gamma + \mu \tag{22}$$

$$\Gamma = \beta \int [(\Gamma' + \mu')/\alpha'] dH(s,s') \tag{23}$$

$$\Gamma q = \beta \int [\Gamma'(x' + q')] dH(s,s') \tag{24}$$

and the complementary slackness conditions: $\mu(m - c) = 0$; $\mu \geq 0$. Note that the envelope theorem has been employed; i.e., Γ is equal to the derivative of the value function with respect to real wealth and μ is equal to the derivative of the value function with respect to real balances. Again, eq. (24) is a standard capital asset pricing equation for z and will not be analyzed here.[6]

Equation (22) implies that if the constraint is binding, the marginal utility of consumption will not equal the marginal utility of real wealth. This result is in contrast to the liquid asset model and represents the major difference between the models from the individual's perspective. In the semi-liquid asset model, augmenting an agent's real wealth by one unit will not lead to a direct increase in utility if the constraint is binding. Only by relaxing both constraints can the agent directly transform wealth into consumption.

Combining eqs. (22) and (23) results in

$$\mu = U_c(c) - \beta \int U_c(c')/\alpha' dH(s,s') \geq 0 \tag{25}$$

By comparing eqs. (25) and (13), the analysis of the statement that the semi-liquid model introduces a precautionary demand for money is made precise. Since the expressions are identical in form, it is obvious that the choice of whether to exit the goods market holding positive nominal balances involves the same marginal tradeoff: the current utility foregone from holding a dollar against the expected utility that a dollar will provide in the following period. In other words, *both* models introduce uncertainty in the decision of whether or not to exhaust money holdings in the goods market. However, there is a difference between the models at the individual level due to the inclusion of nominal balances as an individual state variable in the semi-liquid model. The implication is that agents' optimal consumption (i.e., the policy function obtained as the solution to the dynamic programming problem expressed in eqs. (8) and (20)) will be a function of beginning-of-period nominal balances (M_{t-1}) in the semi-liquid model but not in the liquid model. The expected marginal utility gain that a dollar will provide in the semi-liquid model will, therefore, be affected through two channels: (i) the inflation rate (i.e., the return on money) and (ii) the optimal consumption function. The former effect is also present in the liquid asset approach and generates a store-of-value demand for money. The latter effect, exclusive to the semi-liquid model, can be interpreted as embodying a precautionary demand for money. That is, since consumption is directly affected by beginning-of-period nominal balances, agents may wish to hold cash to ensure consumption next period. Thus the cash-in-advance framework allows a formal characterization of the traditional motives for holding money. The constraint itself obviously implies the transactions motive, whereas money held

6. In a recent paper by Finn, Hoffman, and Schlagenhauf [1990], the Euler equations characterizing optimal equity holdings implied by the two models (i.e., eq. (11) and (24)) are analyzed empirically.

in excess of that needed for consumption purchases may be due to a store-of-value and/or precautionary motive. If the agent's consumption function contains money holdings as a direct argument, then both motives are present whereas, if it does not, only the store-of-value motive is evident. While individual money demand in the liquid asset and semi-liquid asset models does differ, this does not necessarily imply equilibrium money demand will also differ. In fact, as the next section demonstrates, under the common assumption that the economy is populated by homogeneous agents, the behavior of equilibrium real balances will be identical in the two models.

Equilibrium Real Balances

The restriction of analysis to the class of stationary monetary equilibria implies equilibrium real balances will be a function of the state only, i.e., $(M_{t,s}/P_{t,s}) = m(s)$. Additionally, the assumption of a representative agent implies that the agent's first-order conditions will characterize equilibrium when these are evaluated at the market clearing quantities $c_t = x_t$ and $M_t = \overline{M}_t$. In particular, equilibrium real balances are determined by eqs. (6) and (13) for the liquid asset model and eqs. (19) and (25) for the semi-liquid model. In order to obtain expressions in terms of $m(s)$ only, note that the one-period inflation rate for both models can be written as

$$\alpha' = m(s)\ [1 + g\ (s')]/m(s') \tag{29}$$

Then, letting $U_c(s) \equiv U_c[x(s)]$, equilibrium real balances are defined in both models as the solution to the following set of inequalities:

$$m(s) \geq x(s) \tag{30}$$

$$U_c(s) - \beta/m(s) \int U_c(s')m(s') / [1 + g(s')]dH(s, s') \geq 0 \tag{31}$$

(with equality if $m(s) > x(s)$).

Since both models imply the same set of inequalities, the equilibrium real balance function will also be the same. (The proof of the existence and uniqueness of $m(s)$ is contained in the appendix.) To understand the result, note that there are two differences between the models that would, in general, imply different real balance functions: (1) the liquid asset model permits the beginning-of-period distribution of the money stock to be altered by trades in the asset market while the semi-liquid approach does not, and (2) the individual's beginning-of-period money balances are a direct argument of the consumption policy function in the semi-liquid model but not in the liquid model. These differences are eliminated, however, by the use of identical agents because:

(1) Asset prices are sustaining prices so that no trades take place; i.e., the distribution of the money stock is not altered and is the same in both models.

(2) The equilibrium distribution of consumption across agents is the same in both models.

Consequently, the equilibrium real balance function as defined by the separating hyperplane implied by eqs. (30) and (31) is the same. This implies further that only the transactions and store-of-value motives for holding money are manifest in the behavior of equilibrium. That is, since equilibrium consumption for the representative agent is identical in the two models, the fact that nominal balances enter directly into the consumption policy function in one model and do not in another is irrelevant for the characteristics of equilibrium.

III. THE TIMING OF THE MONETARY INJECTION

The proof that the timing of markets has no effect on the equilibrium characteristics of real balances depends critically on all other specifications of the models being identical. In particular, it was assumed in the previous section that the monetary injection was received in the beginning of the period; i.e., in the asset market for the liquid asset model and in the goods market for the semi-liquid model. In Svensson's use of the semi-liquid model, however, he specified that the monetary transfer was also received in the asset market and, hence, at the end of the period.[7] As the introduction stated, this timing convention has the potential to influence the equilibrium behavior of real balances because of the implications for agents' information. Specifically, with the transfer received at the end of the period, the realization of the monetary growth rate at time t provides agents with perfect information about the money stock in the goods market at time t and t+1. Not surprisingly, the enhanced information can influence equilibrium money demand.

Formally, the end-of-period transfer implies two changes in the model specifications analyzed in section II. At the individual level, nominal wealth in period t+1 is now augmented by the period t monetary transfer; hence the last term in eqs. (4) and (17) is lagged one period. (Note that, since the transfers are lump-sum, the necessary conditions characterizing optimal money holdings are unaltered, i.e. eqs. (13) and (25) are still relevant.) At the aggregate level, equilibrium real balances in the goods market at time t are now defined in terms of the period t−1 money stock:

$$\overline{M}_{t-1}/P_t \equiv m(s) \tag{32}$$

As a consequence, the one-period change in the nominal price level between periods t and t+1 is now written as

$$\alpha' = m(s)\,[1 + g(s)]\,/\,m(s') \tag{33}$$

By comparing this expression to its equivalent in section II (eq. (29)), the change in the timing of the monetary transfer is evident. Note that it is the current, as opposed to next period's, monetary growth rate that influences the inflation rate between periods t and t+1. This change reflects the enhanced information agents have about the time path of the relevant money stock (i.e. that available in the goods market) when the transfer is received at the end of the period.

The modification in α' is also present in the set of expressions that define the equilibrium real balance function (the analog to eqs. (30) and (31)):

$$m(s) \geq x(s) \tag{34}$$

$$U_c(s) - \beta/[m(s)(1 + g(s))] \int U_c(s')m(s')dH(s,s') \geq 0 \tag{35}$$

(with equality if $m(s) > x(s)$).

As an illustration of the impact these changes can have, consider the sufficient conditions for dynamic neutrality under the beginning-of-period and end-of-period transfers. That is, when the only source of uncertainty is a stochastic monetary growth rate, the conditions that are sufficient to ensure that real balances are constant, i.e. the inflation rate is always equal to the monetary growth rate. As discussed in Jovanovic [1981], a sufficient condition for a wide class of general equilibrium monetary models is that the mon-

7. Svensson chose this particular timing so that monetary transfers would be as "liquid" as other assets (see his discussion on pg. 923). However, this interpretation does not seem appropriate given that the transfers are lump-sum.

etary growth rate is independently and identically distributed over time. That is, since the current realization of the monetary growth rate provides no information about future growth rates, agents choose the same level of real balances every period.

For precisely this reason, i.i.d. monetary growth rates are indeed a sufficient condition for dynamic neutrality when the transfer is received at the beginning of the period. (This is proven in the appendix.) On the other hand, because of the information provided by the monetary growth rate when the transfer is received at the end of the period, i.i.d. growth rates are no longer sufficient for dynamic neutrality. As demonstrated in the appendix, sufficient conditions in this case are now jointly the absence of serial correlation in the monetary growth rate process and the monetary growth rate takes on non-negative values only. Stated conversely, it is possible for real balances to increase (i.e., the cash-in-advance constraint is not binding) in states with a realization of a negative growth rate. But of course this possibility makes sense given agents' information: a negative realization of g implies that agents know the money stock will fall between the period t and $t+1$ goods markets. Hence, a deflation is expected; if great enough, money will be a viable store of value.

In summary, the assumption that the monetary transfer is received at the end of the period trivializes the information content implied by the stochastic properties of money growth since agents have perfect information about the relevant money stock in periods t and $t+1$. Many of the interesting questions about the implications of monetary uncertainty for equilibrium money demand can not, therefore, be addressed if the end-of-period transfer is used. For instance, the implication of this timing choice is that time-varying conditional means and variances of the monetary growth rate will have no influence on the behavior of equilibrium money demand.

IV. CONCLUSION

The imposition of a cash-in-advance constraint necessitates consideration of the consequences of exogenously specified timing conventions. This paper has analyzed two such timing choices: (i) the timing of markets and (ii) the timing of monetary transfers. With respect to the latter, I demonstrated that beginning-of-period transfers permit a more interesting theoretical role for monetary uncertainty than end-of-period transfers. Hence, for analyses of the effects of monetary uncertainty on equilibrium behavior, the beginning-of-period timing seems more appropriate.

Market timing has potentially important implications for money demand at the individual level, but whether or not these differences manifest in equilibrium aggregate behavior depends on the heterogeneity of agents. In particular, a precautionary demand for money (as defined in the text) can emerge in equilibrium only if agents are heterogeneous. An obvious area for further research is, therefore, the analysis of heterogeneous agent cash-in-advance models and a comparison of equilibrium price level dynamics under different market timing choices.

APPENDIX

1. *Existence and Uniqueness of the Real Balance Function*

The proof of existence and uniqueness of $m(s)$ involves the construction of a contraction mapping. Let $h(s) = U'(s)m(s)$ and assume that $m(s)$ is continuous and bounded so that $h(s)$ is also. Define the operator $Th = h$ by

$$Th = \max\{\beta \int h(s')/[1 + g(s')]\, dH(s,s'),\ U'(s)x(s)\}$$

With the following assumption, it is a straightforward exercise to show that T is a contraction mapping.

$$\text{(A.1)} \qquad \beta \int 1/[1 + g(s')]\, dH(s,s') < 1$$

The reasoning behind (A.1) is illustrated by considering its analog in the non-random, certainty model, $\beta/(1 + g) < 1$, which is a sufficient condition for the existence of a steady-state monetary equilibrium. (See Brock [1975].) In the present model, any steady-state solution would be characterized by an always binding constraint so that the inflation rate will equal the monetary growth rate. Suppose that $\beta/(1 + g) > 1$; then in any proposed equilibrium the real rate of return on money $[1/(1 + g)]$ would be greater than an agent's discount rate $(1/\beta)$. Agents could increase utility by foregoing consumption and acquiring real balances; hence, the proposal could not be a steady-state solution.

To show that T is a contraction mapping, I show that it satisfies Blackwell's sufficiency conditions

(1) T is non-decreasing : true

by inspection

(2) $T(h + c) < T(h) + \beta c$ where c is any constant and $0 < \beta < 1$

Proof: There are three ranges to consider.

$$\text{(I)} \quad \beta \int h(s')/[1 + g(s')]dH < \beta \int (h(s') + c)/[1 + g(s')]dH < U'(s)x(s)$$

$$\text{(II)} \quad U's(s)x(s) < \beta \int h(s'/[1+g(s')]dH < \beta \int (h(s') + c)/[1 + g(s')]dH$$

$$\text{(III)} \quad \beta \int h(s')/[1 + g(s')]dH < U'(s)x(s) < \beta \int (h(s') + c)/[1 + g(s')]dH$$

(I) satisfies the condition by observation. (II) and (III) satisfy (2) because of (A.1). Hence T is a contraction mapping implying the existence of a unique fixpoint $h^*(s)$. The equilibrium real balance function is determined by: $m^*(s) = h^*(s)/U'(s)$.

2. *Sufficient conditions for dynamic neutrality.*

In order to examine the sufficient conditions stated in the text it is assumed that $x(s) = x$ (the only uncertainty is due to the monetary growth rate) with the constant marginal utility of consumption normalized to unity. The equilibrium real balance function in the two models is now defined as the solution to:

(a) Beginning of period transfer

$$\text{(A.2)} \qquad m(s) \geq x(s)$$

$$\text{(A.3)} \qquad m(s) \geq \beta E\{m(s')/[1 + g(s')]\}$$

(b) End of period transfer

$$\text{(A.4)} \qquad m(s) \geq x(s)$$

$$\text{(A.5)} \qquad m(s) \geq \{\beta/[1 + g(s)]\}\, E\{m(s')\}$$

where E{ } denotes the expectations operator.

Equations (A.2) and (A.3) do imply that $m(s) = x$ for all s; that is, money is dynamically neutral when the monetary transfer is received at the beginning of the period. To see this, suppose that the constraint is not binding in a state, denoted as $\hat{s}$, while binding in state $\bar{s}$. Then $m(\hat{s}) > m(\bar{s})$. But eq. (A.3) is an equality in state $\hat{s}$ while a strict inequality in state $\bar{s}$. Since $E\{m(s')/[1 + g(s')]\}$ is a constant due to the assumption that g_t is independently distributed, this results in the contradiction $m(\tilde{s}) > m(\hat{s})$. Hence, $m(s) = x$ for all s.

For the end-of-period transfer, rather than prove $g(s) > 0$ is a sufficient condition directly, I prove the equivalent: if $m(s) > x$ then it is necessary that $g(s) < 0$ for some s. Let $m(\tilde{s}) = \max\{m(s)\colon m(s) > x\}$. Since money is non-neutral by assumption, there is at least one state in which $m(s) > x$ so that $m(\tilde{s})$ is well defined. Eq. (A.5), implies

$$m(\tilde{s}) = \{\beta / [1 + g(\tilde{s})]\}\ \mathrm{E}\{m(s')\}$$

But given the definition of $m(\tilde{s})$, it is also true that

$$m(\tilde{s}) > \mathrm{E}\{m(s')\}$$

which implies

$$\beta / [1 + g(\tilde{s})] > 1 \quad \text{or} \quad g(\tilde{s}) < 0$$

Hence, if $m(s) > x$ for at least one state, then the monetary growth rate must be negative in at least one state also. This implies if $g(s) \geq 0$ for all s, then $m(s) = x$; money is dynamically neutral.

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