Economics 106 – Decision Making
Fall 2006

Lecture Notes: Decision Making under Risk[1]

1 Introduction

- Example: Starting a Business

- 3 difficulties with evaluating risky prospect
  - uncertain prospect with known probabilities

  1. Heterogeneity, range of different outcomes
     - how bad is this really?
       how much worse than?
  2. Relating/Trading Off Probabilities against Probabilities
     - small probabilities
     - how small is 1% risk of going bankrupt
       * intuitive tendency to make little difference between 1/100 and 1/1000; but objective difference huge
       * of major importance in public policy
  3. Complexity: multiple outcomes – multiple prob.
     - chance large that intuitive preference captured by most salient features.

- hence: systematic approach useful – EU theory
  - one of the most useful theories in social sciences.

[1] This is a minimally edited version of my lecture notes. It may not be fully intelligible on its own, but should help you to clarify the content of your own notes and to review the material in your preparation of the final exam.

- spirit of Divide and Conquer:

  break down decision problem in simple steps

  1. Assess probabilities
     - assume done
  2. Assess risks and opportunities via "utility elicitation"
  3. Compute EU
     - in trees: apply avering out and folding back to trees

2 The Key Ideas of Expected Utility Theory

2.1 Definitions

- **utility function** *over final outcomes* $x \in X : u : X \to \mathbf{R}$
  - e.g. $X = R$: dollar gains-losses, ...
- **Expected Utility** of lottery ℓ, which gives x_i with probabilitity p_i :

  $\text{EU}(\ell) = \sum_i p_i u(x_i)$
- DM is a **EU maximizer** if he has utility function u over *final outcomes* x that determines his ranking over lotteries as follows:

$$\ell \succ \text{(resp. } \prec, \sim)\ell' \text{ iff } \sum_i p_i u(x_i) > \text{(resp. } <,=) \sum_i p_i' u(x_i)$$

  - EMVer: $u(x) = x$.

- Two ways to think about utility function:
  - direct – somewhat preliminary
  - as inferred from preferences

2.1.1 Key Idea I: utility cardinal

- cardinal: utility differences measure strength of preference
  - cp in Econ 100: only ordinal ranking meaningful
- e.g. suppose that to you "just as important to avoid 100K loss than to ensure 500K gain"
  - $u(0) - u(-100K) = u(500K) - u(0)$
    - i.e. $u(0) = \frac{1}{2}u(500K) + \frac{1}{2}u(-100K)$
    - 0 utility midpoint
    - *graph*
  - Implies risk-preference :
    - $0 \sim 500K \oplus -100K$
      - · instance risk-aversion
- so: assessment of risk-preferences can be boilded down to assessment ('elicitation') of utility-function
  - dramatically reduces complexity
  - need to elicit a few points, then fill in gaps smoothly
  - can compute EU of any lottery
  - hence also CME:
    $CME(\ell)$ given as the certain amount x such that $u(x) = EU(\ell)$
- technical difficulty:
  only utility differences have meaning, not utility levels
- can pick two numbers arbitrarily and rescale others preserving shape of utility-function
  - change scale in graph
  - u,v generate same risk-preferences iff v obtained from u by addition of constant + positive multiplication
    - e.g. ranking of travel destinations according to u =Fahrenheit gives same result as acc. to v =Celsius.
  - typ. convenient to set $u(Best) = 1$ and $u(Worst) = 0$
    - $Best = W$, $Worst = L$
    - Raiffa: π-indifference curve.
- Where does "utility" assessment come from??
  - "strength of preference" can seem vague

2.1.2 Key idea II: Bootstrapping

elicit utility from own risk-preferences!

- define $\pi-$BRLTS
- $\pi(x)$
  - "Which probability would make me indifferent to winning x"
  - If $u(W) = 1$ and $u(L) = 0$, $\pi(x) = u(x)$
    * hence the π function gives me one particular utility function.

- Again, once π-function determined, all risk preferences determined
  - plug into EU formula

- There are other ways to elicit utility function
  - e.g. **method of utility midpoints** (in homework)
  - Suppose observe $\boxed{0 \sim 500K \oplus -100K}$;
    then $u(0) = \pi(0) = \frac{1}{2}$
  - What x has $u(x) = 0.75$?
    * utility midpoint between 0 and 500?
    1. For which x, $x \sim \boxed{0.75}$
       * i.e. CME($\boxed{0.75}$)?
    2. For which x, $x \sim 500K \oplus 0$; indeed,

$$u(x) = \frac{1}{2}u(500K) + \frac{1}{2}u(0) = \frac{1}{2}1 + \frac{1}{2}\frac{1}{2} = \frac{3}{4}.$$

       * HW
    * note: if EU max., both methods must give same answer!
  - Beauty of this method: translates q about utility differences directly into question about lottery preferences.

- But: we can ask: what sense does it make? *why should I use EU formula*

2.1.3 Key idea III: evaluate complex lotteries using substitution of BRLTs

- In start-up lottery:

0.05	500	1
0.25	100	0.4
0.45	0	0.29
0.1	-50	0.17
0.15	-100	0

  - plot $\pi - function$.
    - fairly risk-tolerant!
  - *substitute* to get ℓ^2_{ST};
    reduce to get $\ell^1_{ST} = \boxed{0.2975}$
    - Equivalent $\pi - BRLT$ better than $\boxed{0.29} = \boxed{\pi\left(0\right)}$,
      hence start-up lottery better than 0
    - CME? IF linear $\frac{0.0075}{0.11} 100K \approx 7K$

- Substitution Principle:

  DM is indifferent to the substitution of any outcome/lottery by another lottery

  - trade-off between outcomes does not depend on other branches of chance tree

- Reduction principle:

  desirability of two-stage lottery depends only on probabilities of final outcomes

- *How can the EU formula know your risk-preferences*
  - about lotteries it has never asked you about?
  - cannot;
    can only under *assumption* that your preferences consistent with SP and RP
    * essentially:
      1. outcomes evaluated independent of other possibilities
      2. probabilities undistorted measuring rod for utilities

- Most peoples preferences fit EU model reasonably well most of the time, but fails sometimes
  - mainly b/c small probabilities.

- Failures of EU model:

  1. Random
     - different ways to elicit utility produce different result
     - not though hard enough/well enough before
       * want to revise some choices after deliberation
  2. Systematic:
     Want to stick to inconsistent choices after deliberation;
     could mean:
     (a) something important left out by EU model
     (b) mistaken intuition

1. A: $4000 for sure B: $ 6000 with 80%, 0 oth.
2. C: [$4000,0.5] vs. D: $ 6000 with 40%, oth.

- If $A \succ B$, then $\pi(4000) > 0.8$
- In that case, $EU(C) > 0.5 \times 0.8 = 0.4 = EU(D)$; i.e. EU maximizers would prefer C over D
  - why not? disappointment – regret
    * include among outcome.

- Argue via SP and RP:
  - say, $\pi(4000) = 0.9$

  1. $C \sim 0.5 \times \boxed{\pi(4000)} + 0.5 \times \boxed{0}$ by SP
  2. $0.5 \times \boxed{\pi(4000)} + 0.5 \times \boxed{0} \sim \boxed{0.5\pi(4000)}$

Does this inconsistency reveal

1. somehting missing from EU model, or
2. mistaken intuition

- Anticipated Disappointment;
  - but: implicit disappointment premium high
  - E.g.: decide for friend

3 Risk Attitudes and the Shape of the Utility Function

- Define: RN, RA, RS via EMV
  - show CME of 50-50 lotteries in 3 cases
  - implied shapes of utility function: concave, linear, convex

- Risk-attitudes need not be the same everywhere
  - Risk-seeking for losses;
    risk-seeking for high gains

- Equivalent in terms of MU:
  - constant
  - decreasing
  - increasing

- MU arguments for risk-preferences
  - overall decreasing DMU
  - e.g. need expensive surgery;
    Bernoulli: prisoner needs 4000 ducats to buy freedom

3.1 Risk-Aversion and Choice of Insurance

E.g. 1% risk of losing L (=\$20,000)

- car totaled
- YES-OR-NO Decision: What is maximal insurance premium that DM is willing to pay for perfect insurance?
  - $\ell = (0.01, -20\ 000; 0.99, 0)$
  - WTP=-CME
    - Since risk-averse, $CME < EMV$
    - *Draw*
    - |WTP-|EMV||="Insurance Premium"
      - e.g. IP=\$500-\$200
      - Can easily be multiple of $|EMV|$

3.2 Risk-Preferences and the Role of Wealth

- two determinants of preferences over gambles:
  - risk/utility attitudes
  - wealth level
- E.g.: No point for Bill Gates to insure car/house (maybe liability)
- In general: reasonable that Insurance Premium for given risk decreases with wealth
  - little graph

3.3 Standard Economic Model: Underlying Preferences over Final Wealth

- start with initial wealth w; add gain/loss x_i; results in "final wealth" $w + x_i$
- postulate vNM utility over **final wealth** U
  - rationale: final wealth determines opportunity set at that state (consumption etc. possibilities; *history should not matter*)
- $u(x)$ at wealth $w = U(w + x)$
- Basic implication:

  risk-neutrality for small gambles
- Dramatic in standard model:

  By putting gains/losses in relation to total life time wealth, only *really large* risks to wealth significant (say 10% plus of wealth!)
  - b/c only over such large difference, MU of final wealth sufficiently variable

3.4 Measing risk-aversion: CRRA

- preference depend only on gains/losses rel. to initial wealth
  - iow.: double stakes + intial wealth– leaves pref. unchanged
  - yields

$$\begin{aligned} U(x) &= \frac{x^{1-\rho}-1}{1-\rho} \text{ if } \rho \neq 1, \text{ and} \\ &= \log x \text{ if } \rho = 1 \end{aligned}$$

  where ρ is "coefficient of relative risk-aversion

    * frequently rescale

- the larger ρ, the larger the insurance premia
  - for smallish risks roughly proportional to ρ
  - CME of $\left(1+d, \frac{1}{2}; 1-d, \frac{1}{2}\right) \approx 1 - \frac{1}{2}\rho d^2$; i.e. "insurance premium"

$$\boxed{\approx \tfrac{1}{2}\rho d^2.}$$

    * Insurance premium on $d = \frac{1}{10} = 10\%$ approx. $\frac{1}{2}\rho\%$.

- Measures equally how fast marginal utility falls:

$$U'(x) = x^{-\rho}$$

  - x +1% implies MU -ρ%

How to determine your ρ?

- Pin down by insurance premium of one lottery
  - pick Q that takes people as far away from loss considerations as possible

- Ex.: **Bill Gates Question:**

  Choice between ∞ with $1-p$ and wealth $w(1-\epsilon)$ with probability p (L_p) indifferent staying put
  - $\infty \quad = \$50$ bio.
  - take ϵ smallish, say $= 10\%$
  - think of wealth loss as "tax" on current assets and all future wage etc. income.
  - Note: elicits π(current wealth)

- think about: 3 different futures with 3 different wealth levels

- Poll:
  1. $p = 10\%$
  2. $p = 30\%$

- If have CRRA utility function, then $\boxed{\rho \approx \frac{p}{\epsilon} + 1}$
  - Economists generally assume that $1 \leq \rho \leq 4$.
  - $\rho \geq 2$ iff $p \geq \epsilon$.

4 The Economic Model of Risk-Taking vs. Prospect Theory

Economic Model:

Expected Utility Theory *over final wealth* with smooth concave utility-function

Prospect Theory:

Preferences *over changes in wealth* with "standard" shapes of value- and weighting function.

Gamble		Economic Model	Prospect Theory	Evidence
Fair (EMV=0)	Large Stakes	RA	RA	RA
	Small Stakes	almost RN	RA	RA
Gains	Mod./Large Probability	RA	RA	RA
	Small Probability	RA	RS	RS (mixed)
Losses	Mod./Large Probability	RA	RS	mixed
	Small Probability	RA	RA	RA

RA: risk-averse;
RN: risk-neutral;
RS: risk-seeking

Basic Departure from Economic Model:
Risk-Aversion to small gambles

- due to Loss Aversion:

  $ lost weighs more heavily than $ gained

Loss Aversion

- Lend 100$ to aquaintance
  1. 90% get back with 20% interest tomorrow
  2. 10% won't get back
- should you lend money?
  -
    $$EMV = 0.9 \times 20 + 0.1 \times -100 = 8$$

- Simplest EU-style explanation:

  utility for gains = $ gained;

  utility for losses = **multiple** of $ lost.

- $u(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x \leq 0 \end{cases}$

- Graph:
  - x = gains – losses
  - Kink at 0
  - Slope =1 for $x > 0$, =λ for $x < 0$.

- Typically value in literature: $\lambda \approx 2$
  - EU with loss aversion: $EU = 0.9 \times 20 + 0.1 \times \boxed{-200} = -2$
  - Note that $\lambda = 2$ means that, for every dollar direct disutility, one $ psychic disutility

- Note: while loss-aversion compatible with EU model (applied to gains/losses), not compatible with Standard Economic Model
  - that u-fu has no kinks
    - nothing special about current wealth (as than that it is current)

- Loss-Aversion closely related to Endowment Effect, Status Quo Bias
  - Cornell coffee mugs

Nominal wage rigidity in Keynesian theory of unemployment

- wage rigidity as a kind of loss aversion
- nominal wage rigidity implies that changes evaluated relative to nominally fixed status quo
  - hence 2% nominal wage reduction perceived as loss,
    while 0% wage increase in presence of 3% inflation is not.
- By consequence, workers accept reductions in real wages more easily when there is inflation
  - Keynesian explanation of “Phillips curve”

Why?

- Spirit of "Heuristics and Biases"
- Heuristics: complex judgment problem
  - here: assessing value of final net asset position
    - essentially: your future life prospects
    - something impossible to do accurately;
      must use 'guesswork'
  - much easier to think about *changes* in position,
    e.g. smallish \$-amount
  - "**Difference Heuristic":**
    assessment and evaluation of levels by assessment and evaluations of differences/changes from reference point

- Organisms employ DHs all the time
  - sensory perception:
    smell/sound primarily of changes

- DH explains
  1. central role of status quo
  2. shape of value function:
     same differences perceived to be smaller, when on top of given change.
     - loss of \$1000 *perceived* to be less than twice as large as \$500 loss.

- *Why loss aversion?*
  - Not clear; literature does not provide answer
  - one suggestion:
    evolutionarily selected heuristic to prevent organism from taking unreasonable risk*s*

- Heuristics lead to **"biases"**
  - if simplification, systematic "errors" to be expected

- Major bias: **framing effects**

"You have been given ..."			
1000	(1000,.50)	(500)*	16:84
2000	(-1000,.50)*	(-500)	69:31

- presented to different groups
- result in same end-state;
  indeed: result in same total change
- in which sense were you given \$1000 in one problem, \$2000 in the other?

- Suggests criticism of risk-seeking with respect to losses

status quo		
0	(-1000,.50)*	(500)
"Suppose you already lost -500"	(-500,.50;+500,.5)	(0)*

- -500 more honest as true current wealth level: that is the EMV you actually face
- moreover, pref. for zero now justifiable in terms of decreasing MU
  - by contrast, first pref. explained via perceptive "shriking" (cognitive error)
- Thus: second preference seems sounder than first

- Critique of Loss Aversion and Risk-Seeking as Biases:
  powerful and provocative
- People's DM may be systematically subrational,
  even though not simply mistake
- here: critique of people's own preferences
  - no "right or wrong"

4.1 Weighting Function

Second departure from EU:
probability weighting (1%≠1%)

To a first approximation:
"Distorted EU" $\sum_i \pi(p_i)\, v(x_i)$

- π non-linear weighting function
- math difficulties in general
- works for "simple lotteries" with at most one gain and one loss.

Example 1:

- Preference for Buying lotteries
  - 1/10M chance at 5,000,000 vs. 1$
  - psychol. weight of 1/10M

- Symmetric: very small probability loss: psychological perception disproportionate to real risk ("fear")

- Show weighting function distilled from many experiments

Example 2:
Russian Roulette

- For which lottery would you pay more money:
  - A) remove 1 bullet out of 4
    B) remover 1 bullet out of 1?

Given amount of ransom R:

- gain $(u(w-R) - u_{dead})$ with prob. $\frac{1}{6}$in both
- lose $u(w) - u(w-R)$ in $\frac{3}{6}$ (in A) or $\frac{5}{6}$ (in B)
- Hence should be willing to pay R in A if willing to pay R in B.

Psychological difference $\pi(\frac{4}{6}) - \pi(\frac{3}{6}) << \pi(\frac{1}{6}) - \pi(\frac{0}{6})$

4.2 Aversion to Small Risks

- 1$ lost more worse than 1$ gained
- People tend not to be consistent when making single decision – multiple independent decisions

- Example: fifty-fifty lottery win 110 vs. lose 90 ($-90 \oplus 110$)
  - Poll
- Compare: 100 gambles simultaneously/sequentially
- Accepting all 100 gambles yields approx. normally distributed with $\mu = 1000$ and $\sigma = 1000$.
  - $P(x \leq 0) \approx 16\%$
  - $P(x \leq -1000) \approx 2.7\%$

Accounted for by loss-aversion:

- linear utility, $\lambda = 2$

- Then

$$
\begin{aligned}
CME(L) &= Ex + (\lambda - 1)\, P(x \leq 0) \times E(x/x \leq 0) \\
&= EMV - LossDiscount
\end{aligned}
$$

- $EU(L_1) = 10 + 0.5 \times -90 = -35$
  - {CME(L_1) $= -17.5$}
- $EU(L_{100}) \approx 1000 + 0.16 \times -300 \approx 950.$
  - =CME(L_{100})
  - Loss Discount 50 cent per gamble!

4.3 Equity Premium Puzzle

- Most investors accumulate money in stock market over long term
  - save during working age from mid 30s to 50s/60s;
    dissave for kids education and retirement
  - thus actual time investment horizon (time lag saving – dissaving) 20-30 years
- Over long term, stocks have outperformed other investments very reliably
  - In U.S. (show graph)
  - Elsewhere also, but less reliably.
- *Equity Premium Puzzle:*
  1. why don't investors put more money in the stock market?
     - this would drive stock prices up and returns down
  2. why is the return/risk premium on stocks as high as it is?

- very large literature
  - in EU, need very large ρ (≥ 10) to explain historical risk premia

Two leading contenders:

1. Subjective Probabilities $\neq$ Historical frequencies
   - "Investor Pessimism"
   - Investors skeptical that great performance in past will be repeated in the future.
2. Additional source of risk-aversion besides DMU
   - Myopic Loss Aversion

Explanation by analogy.

- Investors have short psych. time horizon, and loss averse
- hence: think about choice to invest in stocks as choice over months/year
  - significant probability of loss,
    * leads to big loss discount (as in single gamble)

- Benartzi/Thaler: $\lambda = 2$, horizon = 1 year: explains actual portfolio choices nicely
    - graph

- at the same time:
  if could take long-term point of view, would require much smaller risk premium
  - long-term point of view: count your losses only rarely/at the end
  - show graph

- difference between short-term and long-term risk-premium: "cost of looking"
  -
    - very hard to justify as the value of bad psychological experiences.
      - i.o.w. investors act as if losing money (most likely temporarily) is much more disastrous than it really is
      - Main theme of "Stumbling on Happiness" (Daniel Gilbert)

- Q: how could investor get him-/herself to overcome myopic loss-aversion
  1. Change loss-aversion:
     - think how much worse off you really would be after short-run loss you really would be
     - fret more ex ante than really loose ex post
       - objectively and psychologically
       - "This too shall pass"
     - *difficulty may be that LA secondary adaptation against greed.*
  2. Reframing: Redefine investment horizon psychologically
     - change myopia
  3. Committment: not to look / not to trade!