Granny versus Game Theorist: Ambiguity in Experimental Games*

Jürgen Eichberger
Department of Economics
University of Heidelberg

David Kelsey
Department of Economics
University of Exeter

Burkhard C. Schipper†
Department of Economics
University of California, Davis

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Abstract

We report on an experiment in which subjects choose actions in strategic games with either strategic complements or substitutes against a granny, a game theorist or other subjects. The games are selected in order to test predictions on the comparative statics of equilibrium with respect to changes in strategic ambiguity. We find that subjects face higher ambiguity while playing against the granny than playing against the game theorist if we assume that subjects are ambiguity averse. Moreover, under the same assumption, subjects choose more secure actions in games more prone to ambiguity which is in line with the predictions.

Keywords: Knightian uncertainty, Choquet expected utility, equilibrium under ambiguity, strategic uncertainty, experiments.

JEL-Classifications: C70, C72, C90, C91, D80, D81.

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†Corresponding author: University of California, Davis, Department of Economics, One Shields Avenue, Davis, CA 95616, USA, Email: bcschipper@ucdavis.edu

1 Introduction

In standard game theory, strategic uncertainty in games is resolved in Nash equilibrium, at least for games with a unique Nash equilibrium. Given a player's equilibrium conjecture about opponents' play, she chooses a best response that conforms to the opponents' equilibrium conjecture about her play. What if players lack confidence in their equilibrium conjectures about opponents' play? This is plausible especially if the game is one-shot and players lack previous experience with the same opponents. Lack of confidence in probability judgements is modelled formally by the literature on ambiguity or Knightian uncertainty (Schmeidler 1989, Gilboa and Schmeidler 1989, Bewley 1986). Recently, such approaches have been applied to strategic games (Dow and Werlang 1994, Eichberger and Kelsey 2000, Marinacci 2000).[1] Results on the comparative statics of equilibrium under ambiguity have been derived that should at least in principle be testable (Eichberger and Kelsey 2002, Eichberger, Kelsey and Schipper 2006, Eichberger and Kelsey 2005).

To our knowledge, we present a first attempt to analyze *strategic ambiguity* experimentally. We design an experiment with two-player games, in which we try to introduce ambiguity by varying the identity of the subjects' opponent. Depending on the treatment, subjects have to make choices against a granny, a game theorist or against some fellow subjects. We find more ambiguity averse behavior when subjects face the grandmother compared to the game theorist. However, there does not seem to be a significant difference between behavior against other subjects and behavior against the grandmother.

The main goal of the experiment is a test of results on the comparative statics of equilibrium with respect to changes in ambiguity. In games with strategic complements and positive externalities, equilibrium actions decrease when there is more ambiguity. The same holds for games with strategic substitutes and negative externalities. The intuition is straight forward. As example of the latter class of games, consider a two-person bargaining game. Players face ambiguity over the share of the pie which the opponent will claim. An ambiguity-averse player puts a high weight on bad outcomes, i.e., the event that the opponent demands a large share. As a result, her best-response is to claim a low share. If ambiguity increases, the best-response

[1] See also Epstein (1997), Groes et al. (1998), Haller (2000), Klibanoff (1996), Lo (1996, 1999), Mukerji (1997), and Ryan (2002).

demand decreases.

In experiments, it is difficult to control for a subject's ambiguity. We vary, therefore, cardinal payoffs of a game monotonically keeping the ordinal payoff structure constant. In this way we make games increasingly sensitive to ambiguity-averse behavior. We assume that a decision maker facing ambiguity evaluates an action by the Choquet expect payoff, i.e., she forms expectations with respect to possibly non-additive beliefs. By changing the relative size of cardinal payoffs in a suitable way we can manipulate Choquet expected payoffs such that a given degree of ambiguity has a larger effect on behavior. With this procedure we find that our experimental results are in line with the theoretical predictions for the games we analyze.[2]

The paper is organized as follows: The next section introduces briefly the concept of strategic ambiguity behind our study. Section 3 describes the design of the experiment, followed in Section 4 by a formal statement of hypotheses and the experimental results. The appendix contains a translation of the instructions.

2 Ambiguity in Strategic Games

Consider a finite two-player strategic game $\Gamma = \langle (A_i)_{i=1,2}, (u_i)_{i=1,2} \rangle$ where A_i is player i's finite set of actions and $u_i : A_i \times A_{-i} \longrightarrow \mathbb{R}$ is player i's payoff function. Each player's ambiguity over the opponent's choice of actions is interpreted as a lack of confidence in a probability assessment over opponent's actions. We assume that each player is a Choquet expected utility maximizer. More precise, a player's beliefs are represented by a capacity on A_{-i}, i.e., a real-valued function $\nu_i : 2^{A_{-i}} \longrightarrow \mathbb{R}$ that satisfies monotonicity, for $E, F \subseteq A_{-i}$, $E \subseteq F$ implies $\nu_i(E) \leq \nu_i(F)$, and normalization, $\nu_i(\emptyset) = 0$ and $\nu_i(A_{-i}) = 1$.

In order to compute the Choquet expected payoff given a capacity ν_i, we order the payoffs of each action a_i from highest to lowest, $u_i^1(a_i) > ... > u_i^k(a_i) > ... > u_i^K(a_i)$. Moreover, we denote by $A_{-i}^k(a_i) := \{a_{-i} \in A_{-i} : u_i(a_i, a_{-i}) \geq u_i^k(a_i)\}$ the set of actions of the opponent which yield better payoffs than $u_i^k(a_i)$ with the convention $A_{-i}^0 := \emptyset$. Player i's Choquet expected payoff

[2]Apart from studying ambiguity, our results may be of independent interest for analyzing experimentally to what extent the opponent's identity has a systematic effect on subjects' play in strategic games.

from action a_i given her capacity ν_i is the Choquet integral,[3]

$$U_i(a_i, \nu_i) := \sum_{k=1}^{K} u_i^k(a_i)[\nu_i(A_{-i}^k(a_i)) - \nu_i(A_i^{k-1}(a_i))].$$

Let the support of a capacity supp ν_i be defined as in Dow and Werlang (1994) and Eichberger and Kelsey (2000, 2002). More formally, supp ν_i is defined as the set $E \subseteq A_{-i}$ such that $\nu_i(A_{-i} \setminus E) = 0$ and $\nu_i(F) > 0$ for all F such that $A_{-i} \setminus E \subsetneqq F$. There are several notions of support of a capacity used in the literature.[4] We use this notion here in order to be comparable with the literature cited above.

An *equilibrium under ambiguity* of a finite two-player strategic game Γ is a tuple of capacities $(\nu_i^*)_{i=1,2}$ such that for $i = 1, 2$ there exists a non-empty support supp ν_i^* for which

$$\text{supp } \nu_i^* \subseteq \arg \max_{a_{-i} \in A_{-i}} U_{-i}(a_{-i}, \nu_{-i}^*).$$

This definition is due to Dow and Werlang (1994). In equilibrium under ambiguity, the support of each player's equilibrium capacity is a subset of the opponent's best responses given the opponent's equilibrium capacity. In two-player games, if beliefs are additive, then an equilibrium under ambiguity coincides with a Nash equilibrium.

Capacities can be partially ordered by their ambiguity (see Marinacci 2000, and Eichberger and Kelsey 2002).[5] A game has *strategic complements* (respectively *strategic substitutes*) if there exists an order on the action sets such that each player's best-responses are increasing (respectively decreasing) in the opponent's action a_{-i} on A_{-i}. A game has *positive* (respectively *negative*) *externalities* if there exists an order on the action sets such that $u_i(a_i, a_{-i})$ is increasing (respectively decreasing) in a_{-i} on A_{-i} for all $a_i \in A_i$ and all players.[6] Eichberger and Kelsey (2002, 2005) and Eichberger, Kelsey and Schipper (2006) have shown the following results on the comparative statics of equilibrium with respect

[3]For more on Choquet expected utility theory, see Schmeidler (1989).

[4]For different support notions of capacities compare Haller (2000), Marinacci (2000) and Ryan (2002).

[5]Formally, a capacity ν_i'' is reflects more ambiguity than a capacity ν_i' if for all nonempty $E \subsetneqq A_{-i}$, $\nu_i''(E) + \nu_i''(A_{-i} \setminus E) < \nu_i'(E) + \nu_i'(A_{-i} \setminus E)$.

[6]For games with both properties, we require that those properties use the same order on the action sets.

to ambiguity for players who are ambiguity averse.[7] If a game has strategic complements and positive (respectively negative) externalities, then equilibria under ambiguity are decreasing (respectively, increasing) in ambiguity. The same holds for games with strategic substitutes and negative (respectively, positive) externalities. Moreover, in games with strategic complements and multiple equilibria, sufficient ambiguity selects among equilibria. Rather than reproducing these results formally, we will illustrate them by an example of the class of 3x3 games which we also use in the experiment.

Example Consider the class of 3x3 games

	X	Y	Z
A	c, b	c, c	$c, 0$
B	$0, b$	e, c	$e, 0$
C	a, d	d, c	d, e

with $0 < a < b < c < d < e$. This asymmetric game has a unique pure Nash equilibrium, (B, Y). If we define an order $A < B < C$ and $X < Y < Z$, then it is easy to verify that this asymmetric game has strategic complements and positive externalities. Given a capacity ν, compute the Choquet expected payoffs of the row player for her three actions:

$$\begin{aligned} U(A, \nu) &= c \\ U(B, \nu) &= e\nu(\{Y, Z\}) \\ U(C, \nu) &= a + (d - a)\nu(\{Y, Z\}). \end{aligned}$$

Suppose ν is such that $U(A, \nu) < U(B, \nu)$. Then there exists a more ambiguous capacity ν' with $\nu'(\{Y, Z\}) < \nu(\{Y, Z\})$ such that $U(A, \nu') > U(B, \nu')$. This is the case if and only if $\nu(\{Y, Z\}) > \frac{c}{e} > \nu'(\{Y, Z\}$. So, best-responses are decreasing in ambiguity.

In the experiments we try to manipulate the ambiguity for the same strategic game by letting subjects play against different opponents. There is no theory that tells us how to tie ambiguity to the identity of an opponent. In order to elicit how a given player, faced with the same

[7] Ambiguity aversion is modelled by the Choquet integral of a *convex capacity*. Formally, a capacity is convex if, for all $E, F \subseteq A_{-i}$, $\nu_i(E) + \nu_i(F) \leq \nu_i(E \cup F) + \nu_i(E \cap F)$.

opponent, responds to more ambiguity, we manipulate the cardinal payoffs of games keeping the ordinal payoff structure fixed such that the choice becomes more sensitive to ambiguity. This can be done by manipulating e and c such that $\frac{c}{e}$ changes relative to $\nu(\{Y, Z\})$. The more ambiguity-averse a subject is, the more likely she will choose A rather than B as the ratio $\frac{c}{e}$ falls.[8] □

In the experiment, subjects face three classes of strategic games: Firstly, games with strategic complements, positive externalities and a unique pure Nash equilibrium, henceforth "*strategic complements*", secondly, games with strategic substitutes, negative externalities and a unique pure Nash equilibrium, henceforth "*strategic substitutes*", and thirdly, games with strategic complements and multiple equilibria, henceforth "*multiple equilibria*". There are four 3x3 versions of each class of games for which cardinal payoffs vary monotonically keeping the ordinal payoff structure constant.

Games 1 to 4 in Table 1 are games with strategic complements and positive externalities[9] if we fix the order $A < B < C$ and $X < Y < Z$. They have a unique pure Nash equilibrium, (B, Y). In these games, A is the equilibrium action under ambiguity if ambiguity is sufficiently high, i.e., $\nu(\{Y, Z\})$ is less than the critical value $\frac{c}{e}$. Notice that the ratio $\frac{c}{e}$ increases from game 1 to game 4. The effect of ambiguity on these games has been discussed in the Example.

Games 5 to 8 are games with strategic substitutes and negative externalities if we fix the order $A > B > C$ and $X > Y > Z$. They also have a unique pure Nash equilibrium, (B, Y). For high ambiguity, C is the only equilibrium action under ambiguity. The more ambiguity-averse a subject is, the more likely she will choose C in these games. Since the critical value increases from game 5 to 8, we should observe more subjects choosing C in this order of the games.

Finally, games 9 to 12 are games with strategic complements, positive externalities and multiple equilibria if we fix the order $A < B < C$ and $X < Y < Z$. The pure-strategy Nash equilibria of these games are (A, X) and (C, Z). For a sufficiently high degree of ambiguity,

[8] For the class of games considered above, there is one caveat. We strongly prefer games in which no action is weakly dominated by another (note that ambiguity respects dominance). Thus we also need to increase d whenever we increase c. This influences the evaluation of action C in comparison to action B. However, action A will be preferred to action C.

[9] The identification numbers of the games are in the top left corner of each game matrix.

Table 1: Experimental games

strategic complements

1.	X	Y	Z
A	25, 23	25, 25	25, 0
B	0, 23	100, 25	100, 0
C	3, 27	27, 25	27, 100

2.	X	Y	Z
A	71, 69	71, 71	71, 0
B	0, 69	100, 71	100, 0
C	3, 73	73, 71	73, 100

3.	X	Y	Z
A	86, 84	86, 86	86, 0
B	0, 84	100, 86	100, 0
C	3, 88	88, 86	88, 100

4.	X	Y	Z
A	97, 95	97, 97	97, 0
B	0, 96	100, 97	100, 0
C	3, 99	99, 97	99, 100

strategic substitutes

5.	X	Y	Z
A	3, 3	3, 0	27, 25
B	0, 3	100, 100	100, 25
C	25, 27	25, 100	25, 25

6.	X	Y	Z
A	3, 3	3, 0	72, 70
B	0, 3	100, 100	100, 70
C	70, 72	70, 100	70, 70

7.	X	Y	Z
A	3, 3	3, 0	88, 86
B	0, 3	100, 100	100, 86
C	86, 88	86, 100	86, 86

8.	X	Y	Z
A	3, 3	3, 0	99, 97
B	0, 3	100, 100	100, 97
C	97, 99	97, 100	97, 97

multiple equilibria

9.	X	Y	Z
A	25, 25	25, 0	25, 0
B	0, 25	23, 25	27, 0
C	0, 25	0, 27	100, 100

10.	X	Y	Z
A	70, 70	70, 0	70, 0
B	0, 70	68, 70	72, 0
C	0, 72	0, 72	100, 100

11.	X	Y	Z
A	86, 86	86, 0	86, 0
B	0, 86	84, 86	88, 0
C	0, 88	0, 88	100, 100

12.	X	Y	Z
A	97, 97	97, 0	97, 0
B	0, 97	95, 97	99, 0
C	0, 97	0, 99	100, 100

however, only (A, X) is an equilibrium under ambiguity. Notice that this equilibrium under ambiguity coincides with the Pareto-dominated Nash equilibrium. As one moves from game 9 to 12 the critical value for the choice of the ambiguity-averse action increases.

Table 2 provides the critical values for which ambiguity changes the equilibrium behavior in the twelve games considered.[10]

[10] Numbers with * are just sufficient conditions. For the computations, we took into account the small random constants added (see Footnote 15 in the next section).

Table 2: Sensitivity to ambiguity

Choice	**Capacity**	**Games**			
		strategic complements			
		1	2	3	4
A	$\nu(\{Y,Z\}) <$	0.27	0.72	0.86	0.97
		strategic substitutes			
		5	6	7	8
C	$\nu(\{Y,Z\}) <$	0.27	0.70	0.86	0.94^*
		multiple equilibria			
		9	10	11	12
A	$\nu(\{Y,Z\}) <$	0.27^*	0.71^*	0.86^*	0.97^*

3 Design

The experiment was computerized using z-tree.[11] In each treatment, subjects played the twelve games described in the previous section. For each game, they had to make a choice of action and indicate their belief about the opponent's action. They did not receive any feedback about the opponent's choice of action after each game. We distinguish three treatments:

Treatment gt

In this treatment subjects were asked in each game to make choices of an action twice in the row player's position, one against a grandmother and one against a game theorist.[12] Both, the granny and the game theorist, were real people.[13] Their choices were recorded prior to the experiment with a paper-based questionnaire. The subjects knew that. Both, the granny and the game theorist took the column player's position. They knew that they were playing

[11] We are grateful to Urs Fischbacher for making this experimental software available to the profession (Fischbacher 1999).

[12] For a screen-shot of this treatment see Figure 4 in the appendix.

[13] We want to emphasize that our experiment did not involve any deception of subjects. All the information about the granny and the game theorist provided to the subjects were true.

against subjects from subject-pool of the Bonn Laboratory for Experimental Economics (mostly students). Until the very end of the experiment, subjects did not know either the choices of the game theorist or the granny.

In addition to making choices, each subject was asked to state which actions of the respective opponent she did "take into consideration for her choice". The answers to this question provided us with information about the strategies of the opponent, which subjects believed to be relevant for their choice. We take these "stated beliefs" as a proxy for the support of the subjects' beliefs about their opponents' behavior.

Treatment g

In this treatment subjects played only against the grandmother. Hence, they had to make only one choice of action in each game. Otherwise this treatment is identical to Treatment **gt.**

Treatment s

In Treatment **s** subjects were playing against each other. An equal number of subjects was selected for the row player position and the column player position. In each game, each subject made a single choice of action against another subject. Subjects did not know the identity of the opponent. For computing payoffs, players were matched randomly with an opponent. In all other respects this treatment is identical to the Treatments **gt** and **g**.

For our method of testing ambiguity, we need to assume that ambiguity does not change during the experiment. Hence, special efforts were undertaken in order to avoid learning effects. First, we provided no feedback about the opponent's choices between games. Second, we made comparisons between games difficult. We feared that if similarities of ordinal payoffs are recognized, subjects analyze the games only a few times and then "log in" to a particular default action. Subjects could not compare the games by clicking back and forward. They faced the games in a random order.[14] Moreover, the games' payoff structure was disguised by adding

[14]The games were presented in the following order: 2, 7, 9, 4, 6, 1, 11, 8, 12, 3, 5, 10.

a randomly chosen small positive constant to each player's payoff.[15] These constants perturb the cardinal payoffs slightly, they make the games asymmetric but keep the ordinal payoffs constant. In addition, subjects had to solve a payoff-irrelevant memory task between games. For this task, they had to memorize a couple of digits displayed for 5 seconds and repeat them on the next screen. There is evidence in experimental cognitive psychology (MILLER 1956) that humans' short-term memory span is limited to a few digits only. With this memory task we wanted to erase the short-memory of previous games, thus making comparisons more difficult.

Prior to the experiment, subjects received written instructions in German in which the experimental setting was explained in detail (see the appendix for a translation). According to the instructions, subjects knew that they were to make choices in 12 games against a *granny*, a *game theorist*, or *other subjects*, respectively. Subjects were, however, not informed about the types of games which they were to play. In order to be convincing in our claim that the grandmother and the game theorist were indeed real people we provided subjects with additional information about their background. E.g., we informed subjects that the granny is old, raised 8 children, and lives in a village in East-Germany and that the game theorist is a successful professor.

The instructions contained also an example of a game which did not belong to the classes of games which they would face in the experiment. With this example we tested prior to the experiment whether subjects understood how payoffs in a game are derived given the choices of the players. The instructions contained also the exchange rate, for which payoffs where exchanged into EURO at the end of the experiment.

At the end of the experiment, subjects had to fill in a brief questionnaire at the computer. The questionnaire contained questions about profession, gender, prior knowledge of game theory or economics, as well as how ambiguous they felt about opponents' choices. Subjects did not know the questions of the questionnaire when they played the games.

[15] The following table contains the constants which were added.

	Game											
Player	1	2	3	4	5	6	7	8	9	10	11	12
Row	3	2	1	0	3	1	2	1	3	2	1	0
Column	1	1	0	1	1	2	1	3	0	1	2	1

Once all the information was collected, three games were randomly selected, their outcomes were computed, converted into EUROS, and paid to the subjects immediately after the experiment. The same holds for the granny and the game theorist, except that they received their payoffs several days after their choices. The subjects' answers to our questions were not rewarded. The experiment lasted for approximately half an hour and subjects earned on average EUR 10.50.

The participants of our experiment were 54 subjects from the subject-pool of the Bonn Laboratory for Experimental Economics, a grandmother and a game theorist. All, but one, subjects of the Bonn Laboratory reported that they were students. About 36 percent were students of economics or mathematics. Approximately 24 percent had participated in a course on game theory. Of the students, 36 percent were female.

The granny and the game theorist were approached directly by the experimenter. We collected the data from the granny and the game theorist a couple of days prior to the experiment. The students' experiment was conducted in the Bonn Laboratory for Experimental Economics in June 2004.

We had 18 subjects for each treatment. Since choices were not revealed until the very end of the experiment, we have 18 independent observations for each treatment. Because the games are not symmetric, however, only the 9 observations of the row players in Treatment **s** are comparable with the observations from the other treatments.

4 Hypotheses and Results

Ambiguity about the behavior of the opponent as modelled by the Choquet expected utility approach described in Section 2 induces predictable behavior in games. Our first set of hypotheses and results concern the question whether there are any measurable effects of ambiguity about the opponent's behavior. Our second set of hypotheses and results deals with these comparative statics predictions.

We know from previous experiments on ambiguity in single person decision problems (Camerer and Weber 1992) that the majority of subjects behave in an ambiguity-averse manner. Hence, we maintain the assumption that subjects behave ambiguity-averse throughout this experiment.

4.1 Is there ambiguity?

Our motivation for treatments with a grandmother and a game theorist comes from the fact that behavior of subjects may in general not be ambiguous enough to produce observable effects. A priori it is not clear why the behavior of a grandmother should be more ambiguous than that of a game theorist. Given the subject pool of students at the University of Bonn, however, who in some cases have had some experience with game theory, our presumption was that these students would feel less ambiguous about the behavior of an expert game theorist than about the behavior of the grandmother, obviously an non-specialist opponent. We tried to re-enforce this "non-specialist" feature of the grandmother by explicitly mentioning in the instructions that the granny, in contrast to the game theorist, had difficulties in understanding the experimental set up.

Based on this assumption we expect that subjects felt more ambiguity playing against the granny than playing against the game theorist in Treatment **gt**, the only treatment where such a direct comparison is possible. Our experimental results provide us both with a subject's self-assessed feeling of ambiguity and with her actual choice of action. This design motivates the following two hypotheses.

Firstly, we consider ambiguity associated with the player. We predict that subjects will report more ambiguity when playing against the granny. Secondly, we look at ambiguity about the opponent's choice of action. We predict that the higher ambiguity regarding the granny's choice is reflected in the stated beliefs about the set of the opponent's actions which a subject considers possible. The more actions of the opponent a player takes into account, the more ambiguity she experiences. Hence, beliefs about the grandmother's choice should be more coarse than beliefs about the game theorist's choice.

Hypothesis 1 *In Treatment **gt**,*

(i) subjects report more ambiguity about the behavior of the granny than about the behavior of the game theorist,

(ii) stated beliefs about the grandmother's choice of actions are coarser as the stated beliefs about the game theorist's actions.

Secondly, regarding actual behavior, we predict that more subjects choose the more ambiguity-averse action if they face the grandmother. For games with strategic complements (Games 1 to 4), A is an equilibrium action under ambiguity and, for games with strategic substitutes (Games 5 to 8), C is an equilibrium action under ambiguity, while B is the unique Nash equilibrium in both cases. In games with multiple equilibria (Games 9 to 12), actions A and C are Nash equilibrium actions. For high ambiguity, however, only A remains an equilibrium action under ambiguity. We should, therefore, expect that the equilibrium actions under ambiguity, A in Games 1 to 4 and C in Games 5 to 8, will be chosen more often against the granny than against the game theorist. Moreover, we would expect to see the unique Nash equilibrium strategy B in Games 1 to 8 and the Pareto-dominant Nash equilibrium strategy C in Games 9 to 12 more often played against the game theorist than against the granny.

Hypothesis 2 *In Treatment **gt**, we expect to observe the following behavior:*

(i) In games with strategic complements (Games 1 to 4), more (respectively, less) often action A (respectively, B) is chosen against the granny than against the game theorist.

(ii) In games with strategic substitutes (Games 5 to 8), more (respectively, less) often action C (respectively, B) is chosen against the granny than against the game theorist.

(iii) In games with multiple equilibria (Games 9 to 12), more (respectively, less) often action A (respectively, C) is chosen against the granny than against the game theorist.

Turning to our results. In the questionnaire of Treatment **gt** we asked subjects the questions listed in Table 3. These questions relate to the ambiguity associated with the opponent's identity. Table 3 shows that 72% of the subjects feel they can predict the behavior of the game theorist better than that of the grandmother. Consistent with this assessment, 72% of the subjects report that they prefer to play against the game theorist. We can reject the hypothesis that subjects can guess the granny's behavior better than the game theorist's behavior (resp. prefer to play against the granny than against the game theorist) at the 0.05 confidence level using a Binomial test. For the third question, the degree of certainty was measured on an integer scale ranging from 0 to 5, with *complete uncertainty* at 0 and *complete certainty* at 5. Table 3 reveals that, on average, subjects were more certain about the game theorist's behavior with 3.3 than about the grandmother's behavior with 1.6. These averages rather hide the actual extent of the uncertainty, since 10 of 18 subjects were certain or completely certain (4 or 5) about the behavior of the game theorist and just 2 subjects felt uncertain or very uncertain (0 or 1). Even stronger is the rating of the granny, where only one subject was certain or completely certain (4 or 5) about the behavior of the granny and 10 subjects felt uncertain or very uncertain (0 or 1). Using a Wilcoxon Signed Ranks test, we can reject the hypothesis that subjects can guess the behavior of both opponents equally well at the 0.03 confidence level.

Table 3: Perceived ambiguity

Question	**Game theorist**	**Granny**
1. Whose behavior can you guess better?	72%	28%
2. Whom would you prefer to play against?	72%	28%
3. How certain are you about the behavior of ...?	3.3	1.6

To see which of the opponent's strategies subjects considered as important for their choice, we turn to Figure 1. This figure shows how often subjects reported a non-singleton belief about the opponent's actions. Clearly, stated beliefs differ by opponents. Subjects in Treatment **gt** state more often a non-singleton belief[16] when playing against the grandmother (50 percent) than when playing against the game theorist (40 percent). This provides some support for our hypothesis that subjects feel more certain about the behavior of the game theorist. We can reject the hypothesis that subjects stated equally often a coarse belief (i.e., two or more actions) for the game theorist and for the granny at a 0.05 confidence level using a Wilcoxon Signed Rank test.

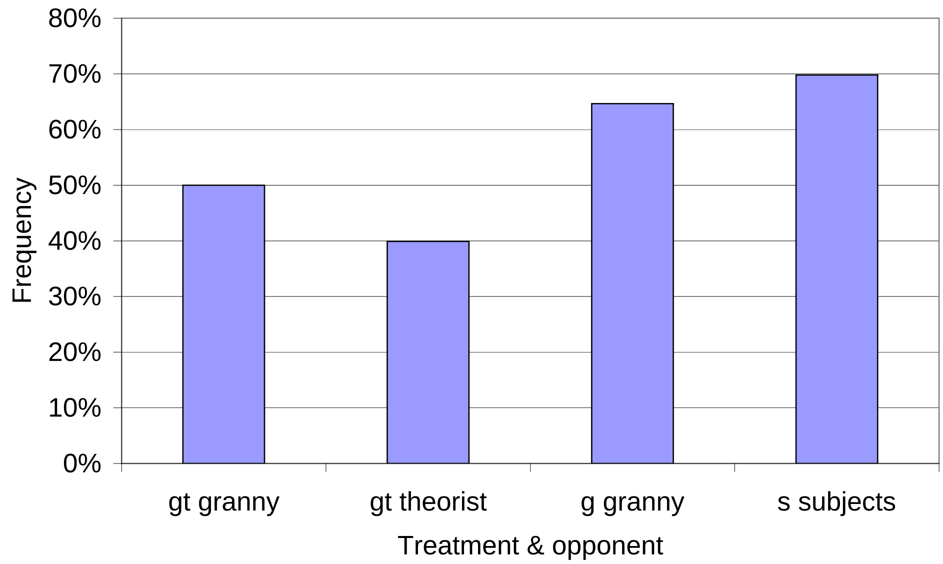
We summarize these results in Observation 1.

Observation 1 *In Treatment* ***gt****, we can not reject Hypothesis 1:*

(i) Subjects reported significantly more ambiguity about the behavior of the granny than about the behavior of the game theorist.

[16]These averages are calculated for all subjects who stated a belief. In four percent of the cases subjects did not state a belief at all

Figure 1: Non-singleton support of stated beliefs



(ii) *Stated beliefs about the grandmother's choice are significantly more often coarser than stated beliefs about the game theorist's choice.*

Figure 2 provides information on Hypothesis 2. In the games with strategic complements, in the upper diagram of Figure 2, 36 percent of subjects chose the equilibrium action under ambiguity A against the grandmother, while only 21 percent chose this action against the game theorist. This difference is significant at a 0.11 level using a Wilcoxon Sign Rank test. This observation is consistent with Hypothesis 2 (i). However, also the unique Nash equilibrium action B is chosen more often against the grandmother (by 39 percent of the subjects) than against the game theorist (by 35 percent of the subjects), which is contrary to Hypothesis 2(i). This difference though is insignificant (0.38 level).[17] It is surprising how often action C, which is neither a Nash equilibrium action nor an equilibrium action under ambiguity, was chosen against the game theorist (by 44 percent of the subjects) in this sequence of games.[18]

[17]We have to note a caveat: Since Treatment **gt** concerns a one sample treatment with dependent variables, we could not test for the difference of the joint distributions of A's and B's.

[18]We suspect that this obviously "irrational" behavior against the game theorist may be a consequence of the random order of games, since the choice of C was most pronounced in games following a multi-equilibria game

In the games with strategic substitutes in the middle diagram of Figure 2, action C is chosen more often against the grandmother (by 28 percent of subjects) than against the game theorist (by 18 percent of subjects), which is significant at a 0.10 level. Strategy C is the only equilibrium action under sufficiently large ambiguity. The unique Nash equilibrium action B is chosen less often against the grandmother (by 68 percent of subjects) than against the game theorist (by 78 percent of subjects). This difference is only significant at a 0.12 level. Both observations are consistent with Hypothesis 2(ii).

Finally, in the games with multiple equilibria in the lower diagram of Figure 2, 55 percent of the subjects chose action A against the grandmother, while only 33 percent of the subjects chose this strategy against the game theorist. This is significant at a 0.08 level. In contrast, action C is chosen less often against the grandmother (by 47 percent of the subjects) than against the game theorist (by 58 percent of the subjects). This difference is not significant (0.24 level). For low ambiguity, both actions are Nash equilibrium actions and actions in an equilibrium under ambiguity, but if ambiguity is sufficiently large then action A becomes the unique equilibrium action under ambiguity. Both observations are consistent with Hypothesis 2(iii).

Observation 2 summarizes these findings.

Observation 2 *In Treatment* ***gt****, there is mixed evidence for Hypothesis 2(i), but we cannot reject Hypothesis 2(ii) and Hypothesis 2 (iii):*

*(i) In Games 1 to 4 (*strategic complements*), subjects chose significantly (resp. insignificantly) more often the ambiguity averse action (resp. the Nash equilibrium action) against the grandmother than against the game theorist.*

*(ii) In Games 5 to 8 (*strategic substitutes*), subjects chose significantly more (resp. less) often the ambiguity averse (resp. Nash equilibrium) action against the grandmother than against the game theorist.*

*(iii) In Games 9 to 12 (*multiple equilibria*), subjects chose significantly more (resp. insignificantly less) often the ambiguity averse action (resp. the Pareto-dominant Nash equilibrium) against the grandmother than against the game theorist.*

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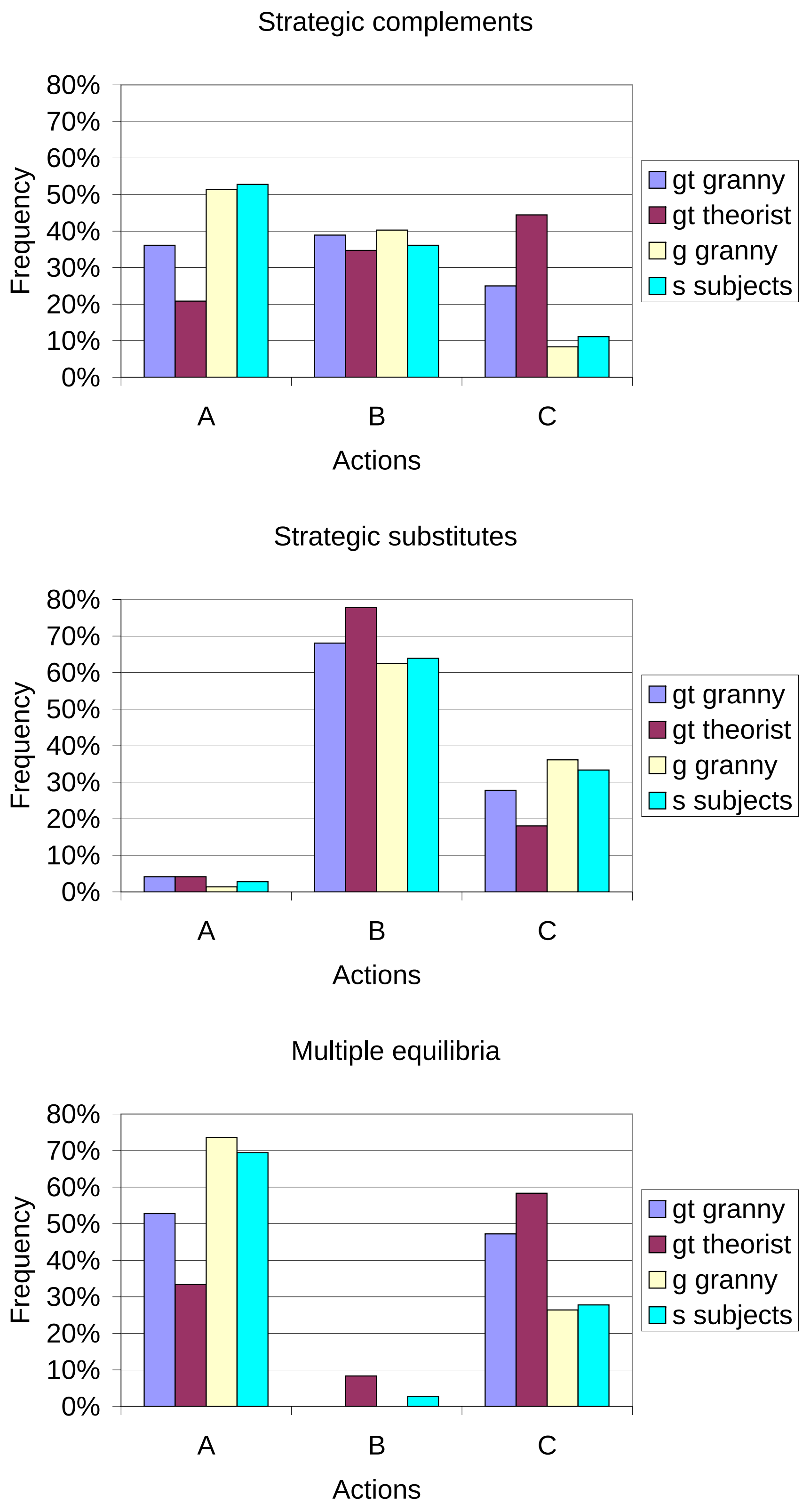
where C was the equilibrium action of the Pareto-dominant Nash equilibrium.

Figure 2: Distribution of actions



Treatment **gt** provides the opportunity to compare the behavior of subjects playing games against two opponents with *identical payoffs* but clearly *distinguished by personal characteristics.* The two other treatments, Treatment **g** and Treatment **s**, serve as control treatments. Our a priori hypothesis was that behavior when playing against other subjects should create less ambiguity than playing against the granny, but more ambiguity than playing against the game theorist. Comparing firstly Treatments **g** and **s,** these considerations can be expressed by the following two hypotheses. The first two hypotheses compare stated beliefs and actual behavior in Treatment **g** and in Treatment **s**. Hypothesis 3 is analogous to Hypothesis 1. We assume that the higher ambiguity against the granny compared to the other subjects is reflected in the subjects' statements.

Hypothesis 3 *In comparing Treatment **g** and Treatment **s**, stated beliefs about the grandmother's choice are more coarse compared to stated beliefs about other subjects.*

Hypothesis 4 parallels Hypothesis 2. We predict that the equilibrium action under ambiguity will be chosen more often against the granny than against other subjects, while the Nash equilibrium action will be more often against other subjects.

Hypothesis 4 *In comparing Treatment **g** and Treatment **s**, we expect the following facts:*

(i) In games with strategic complements *(Games 1 to 4), more (respectively, less) often action A (respectively, B) is chosen against the granny than against other subjects.*

(ii) In games with strategic substitutes *(Games 5 to 8), more (respectively, less) often action C (respectively, B) is chosen against the granny than against other subjects.*

(iii) In games with multiple equilibria *(Games 9 to 12), more (respectively, less) often action A (respectively, C) is chosen against the granny than against other subjects.*

The answers to the questionnaire at the end of the experiment hint a first answer to Hypothesis 3. In Treatments **g** and **s**, we asked each subject to rate on a scale from 0 (complete uncertainty) to 5 (complete certainty) how certain he or she was about the behavior of the grandmother or the other subject, respectively. The average reports are very similar in both treatments, 2.6 for Treatment **g** and 2.7 for Treatment **s**.

A similar conclusion can be drawn when looking at stated beliefs in Figure 1. In fact, there was more ambiguity reported about the choices of the other subject than about those of the granny. However, the difference is not significant (0.27 level using a Wilcoxon-Mann-Whitney test). Observation 3 states this result.

Observation 3 *We can reject Hypothesis 3: Stated beliefs about the grandmother's choice in Treatment **g** are less coarse than stated beliefs about other subject's choice in Treatment **s**.*

Examining Figure 2 shows that also the choices of actions were almost identical in both treatments. In the case of strategic complements (Games 1 to 4), the equilibrium action under ambiguity was chosen even slightly more often against the other subject than against the granny and the Nash equilibrium action was chosen more often against the granny. The difference between the joint distributions of actions is not significant (0.9 level for strategic complements and substitutes and 0.5 for multiple equilibria using a $\mathcal{X}^2$ test).

Observation 4 *In comparing Treatments **g** and **s**, we can reject Hypothesis 4:*

(i) In games with strategic complements *(Games 1 to 4), insignificantly more (resp. less) often action B (resp. A) is chosen against the granny than against other subjects.*

(ii) In games with strategic substitutes *(Games 5 to 8), insignificantly more (resp. less) often action C (resp. B) is chosen against the granny than against other subjects.*

(iii) In games with multiple equilibria *(Games 9 to 12), insignificantly more (resp. less) often action A (resp. C) is chosen against the granny than against other subjects*

Observations 3 and 4 suggest that the perceived ambiguity as well as the actual behavior were similar in Treatment **g** and Treatment **s**. It is important to keep in mind, however, that, for Treatment **s**, this comparison rests on a much smaller number of observations, since only the behavior of the nine row players are used.

Finally, comparing Treatment **gt** and **g**, our ex-ante presumption was that one would find the same perceived ambiguity and the same behavior under ambiguity in regard to the granny in both treatments.

Hypothesis 5 *Choices and stated beliefs when playing against the grandmother in Treatment **gt** do not differ from Treatment **g**.*

In fact, Figure 1 shows quite clearly that subjects were considering significantly more often non-singleton beliefs when facing the grandmother in Treatment **g** than in Treatment **gt**. This difference is significant at a 0.03 level using a Wilcoxon-Mann-Whitney test. Similarly, Figure 2 reveals that the ambiguity-related actions, A in Games 1 to 4, C in Games 5 to 8, and A in Games 9 to 12, were chosen more often in Treatment **g** than in Treatment **gt**. The difference between the joint distributions of actions is significant at a 0.05 (resp. 0.02) level for strategic complements (resp. multiple equilibria) but insignificant for strategic substitutes (0.5 level with a $\mathcal{X}^2$ test). To sum up, it appears that subjects felt more ambiguity when playing against the grandmother in Treatment **g** than in Treatment **gt**.

Observation 5 *In comparing Treatments **gt** and **g**, we can reject Hypothesis 5:*

*(i) Stated beliefs about the grandmother's choice are significantly more often coarser when playing against the grandmother in Treatment **g** than in Treatment **gt**.*

*(ii) Play against the granny in Treatment **gt** differed significantly from Treatment **g**. In particular, the ambiguity-related actions (resp. Nash equilibrium actions) were more (resp. less) often chosen in Treatment **g** than in Treatment **gt**.*

Observation 5 records stronger ambiguity effects in Treatment **g** than in Treatment **gt**. Though we did expect that playing against the grandmother would create some ambiguity, we were surprised to find this ambiguity to be substantially smaller in the Treatment **gt** where subjects face both the granny and the game theorist. We speculate that this finding is due to a presentation effect. Treatment **gt** is likely to lead subjects towards a comparative judgement between the game theorist and the granny. Such comparative analysis may lead to different judgements of the granny when the granny is the only opponent to judge as in Treatment **g**.

4.2 How do subjects react to ambiguity?

The core hypotheses of this article concern the comparative statics analysis of behavior under ambiguity. As explained in Section 2, we constructed the sequence of games in each of the three variants, *strategic complements, strategic substitutes* and *multiple equilibria*, such that the critical level for changing behavior towards the equilibrium action under ambiguity rose with the number of the game. Table 2 contains these critical levels. In each group of games the sensitivity to ambiguity increased with the number of the games. Hence, we advance the following hypothesis.

Hypothesis 6 *For all treatments, we expect to observe following comparative statics:*

(i) In games with strategic complements*, choices of action A (respectively, B) increase (respectively, decrease) from Game 1 to 4.*

(ii) In games with strategic substitutes*, choices of action C (respectively, B) increase (respectively, decrease) from Game 5 to 8.*

(iii) In games with multiple equilibria*, choices of action A (respectively, C) increase (respectively, decrease) from Game 9 to 12.*

Turning now to our results, the left diagrams of Figure 3 show how the frequency of the equilibrium action under ambiguity changes in all treatments and against all opponents. With the exception of Treatment **gt**, we find for each class of games that the equilibrium action under ambiguity increases as the games become more ambiguity-sensitive, i.e., from the lower to the higher game number. The exception is play against the game theorist in games with strategic complements and against the granny in games with strategic substitutes, where in Games 3, 8 and 10 a decline has to be noted.

The right diagrams in Figure 3 show the frequency of choice for the unique Nash equilibrium action B in response to increasingly ambiguity-sensitive games. With the exception of Games 3, 8, and 11 we observe a decrease in all treatments and against all opponents.[19]

We test the results on the comparative statics by comparing each subject's choices in Game 1 versus Game 4 (and similarly Game 5 vs. Game 8 and Game 9 vs. Game 12). We exclude all observations of actions that are neither an equilibrium action under ambiguity nor a Nash equilibrium. We test as the null-hypothesis that switches from the Nash equilibrium to the

[19]Notice that the exceptions seem to occur in the same games.

ambiguity-averse action and vice versa are equally likely. We can reject this hypothesis except for play against the game theorist in games with strategic complements and in games multiple equilibria in Treatment **gt**.[20] Summarizing these results, we obtain Observation 6.

Observation 6 *We can not reject Hypothesis 6, except for play against the game theorist in games with strategic complements and multiple equilibria (Treatment **gt**).*

5 Concluding Discussion

In experiments on single-person decision problems, ambiguity plays a role as many studies of the Ellsberg-paradox show. Camerer and Weber (1992) provide a survey of these results. Strategic problems are usually even more complex, so it appears reasonable to assume that ambiguity plays an even bigger role in strategic games. Camerer and Karjalainen (1994) report experiments on strategic versions of Ellsberg's two and three color experiments which seems to confirm this presumption. In their experiments ambiguity concerns the payoffs of the opponents. They find evidence that a substantial fraction of behavior is inconsistent with the assumption of additive beliefs over opponents' types.

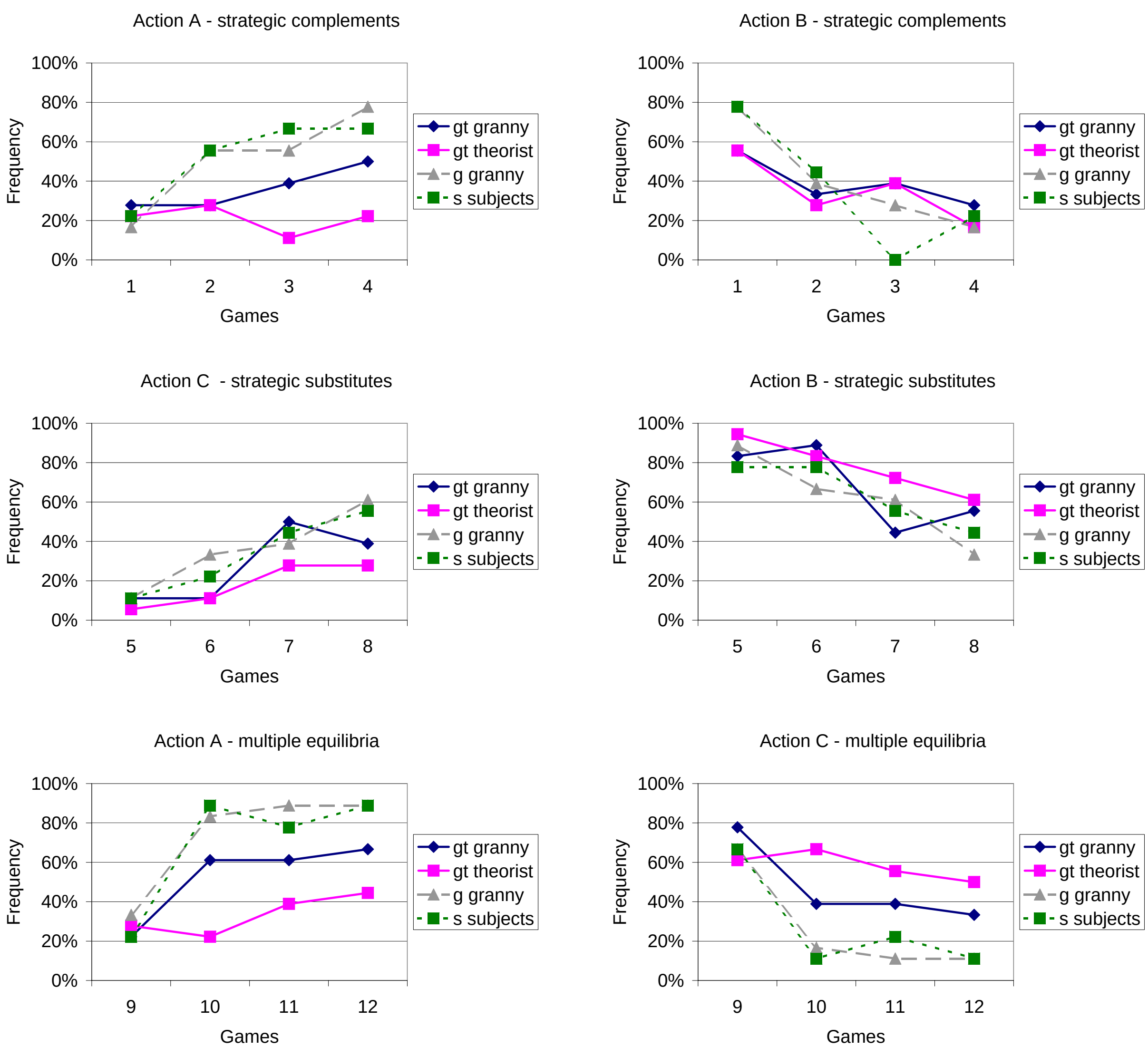
To our knowledge, we present a first attempt to analyze *strategic ambiguity* experimentally. By varying the identity of the opponent, we try to introduce different levels of ambiguity in strategic games. Moreover, by varying the cardinal payoffs but keeping the ordinal payoff structure constant, we make games more or less sensitive to the given amount of ambiguity in the experiment. We find that both varying opponents and varying the payoff structure have effects predicted by the theory on ambiguity in games.

In Treatments **gt** and **g**, we used "loaded" instructions in the sense that we described the

[20] The significance levels using a Sign Test are given by

Treatment	**strategic complements**	**strategic substitutes**	**multiple equilibria**
gt granny	0.03	0.02	0.01
gt game theorist	0.25	0.11	0.23
g	0.01	0.01	0.01
s	0.03	0.12	0.03

Figure 3: Comparative statics



background of the granny and the game theorist in order to be convincing in our claim that these opponents were indeed real persons. It is therefore justified to ask whether social motives could have brought about the observed difference in choices against the granny and the game theorist. A preliminary check suggests that social motives such as altruism or inequity aversion will induce behavior which is opposed to the one predicted by ambiguity aversion. Thus, they may in fact strengthen our comparative statics results.

A Example of Instructions: Treatment gt (Translation)

Welcome to the Experiment

You participate in an experiment on decision making. You can earn some money. Your earnings depend on your decisions as well as the decisions of a grandmother and a game theorist. Latter decisions we recorded already prior to the experiment today.

The Grandmother

The grandmother is 84 years old. She lives beside a forest in a village in Saxony. She comes from a farmer's family and raised 8 children. She likes to take care of her large garden, to solve crossword puzzles, and to watch TV. She faced some difficulties with understanding today's experiment.

The Game Theorist

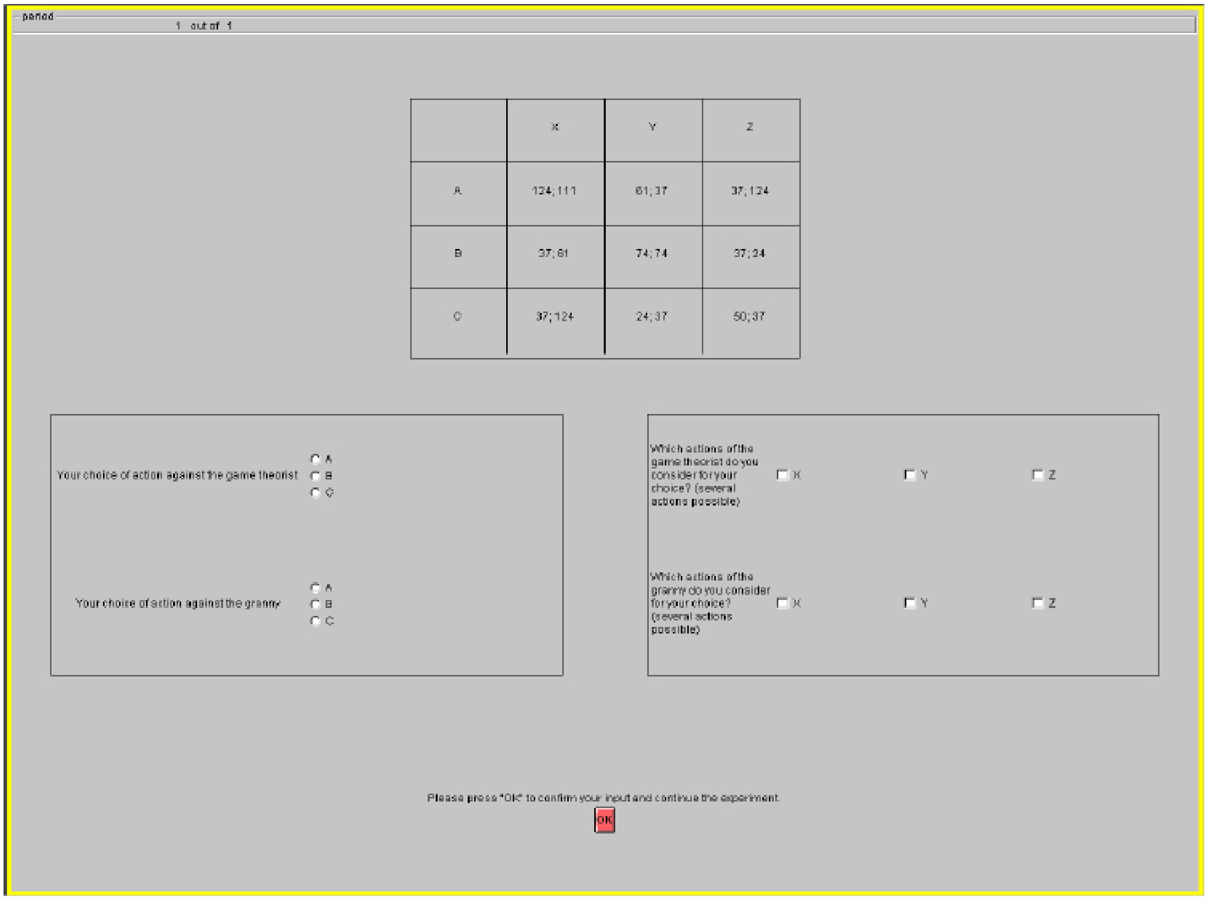
Game theory is a mathematical theory of strategic decision making such as today's experiment on decision making. The game theorist is Professor of Economic Theory at the University of Bonn. Previously, he worked at Stanford University and Humboldt University, Berlin. He earned a diploma in mathematics and a Ph.D. in economic theory. He published quite a number of articles on game theory in international journals such as the Journal of Economic Theory. He didn't face any difficulties with understanding today's experiment.

Your Decision

Your goal is to maximize your earnings through your choice. You will face decision problems like in the following example (Figure 4):

You have three actions (A, B, and C), which are marked as rows in above table. The other participant (the grandmother or the game theorist) has three actions as well (X, Y, and Z) (the columns in above table). The numbers in the cells of the table indicate the possible payoffs, whereby your payoff is always

Figure 4: Screen-Shot



the first number in front of the semicolon (;) of each cell, whereas the second payoff belongs to the other participant. For example, if you choose A and the other participant chooses Y, then you receive 56 Taler and the other participant 99 Taler.[21]

Under the table to the left you are supposed to choose your action: One action against the grandmother and one against the game theorist. Prior to your decision, we naturally do not inform you, how the grandmother and the game theorist chose against you. Your payoff depends as indicated in above table on your choice and the choice by the grandmother and the game theorist.

Under the table to the right you are supposed to mark the actions that you can not rule out for the

[21]Note that Figure 4 contains a translated screen-shot in which numbers do not correspond to the translation of the instruction. This is not the case in the German original.

grandmother and the game theorist. These are the actions for which you assume that they could be eventually chosen by the grandmother or the game theorist. Here it is possible to mark several actions.

After you made your selection, click "O.K.", and the experiment is continued with a memory task on a new screen. The memory task does not influence your payoff but serves just as an intermediate step between the decision making situations. A sequence of numbers is displayed to you for 5 seconds, which you should try to remember. After 5 seconds you are asked on a new screen to reproduce the sequence. After the memory task a new screen appears with a new decision making situation analogous to above. In total there are 12 decision making situations.

Your Earnings

After the decision making situations follows a brief questionnaire. Then you will be informed about your total earnings. To calculate your total earnings, 3 decision making situations are selected randomly. For each of these 3 decision making situations the payoff depends on your decisions and the decision of the grandmother and the game theorist as described above. Your total earnings is the sum of payoffs from the 3 decision making situations against the grandmother as well as the 3 decision making situations against the game theorist. Your total earnings are exchanged with an exchange rate of 40 Taler = 1 EUR. This amount will be paid to you immediately after the experiment in cash.

In each cabin is a exercise-sheet, which should be completed before the experiment, and which will collected by the experimenter. Only then the experiment will be started. If you have questions now or during the experiment, please quietly contact the experimenter.

Thank you for your participation.

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