Confidence in the Prior and Reverse Bayesianism

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Abstract

In this note, we show that when the agent lacks a tiny bit of confidence in her prior, she may not appear to be reverse Bayesian upon becoming aware of an event. This suggests an extension of reverse Bayesianism that allows also for dismissal of the prior upon becoming aware. An agent has a second-order belief over priors measuring her confidence over priors and picks the prior with highest second-order probability similar to Ortoleva (2012). Upon becoming aware, she updates all of her priors as well as her second-order belief by reverse Bayesianism, and picks the one with highest updated second-order probability. Such an updating rule may be more appropriate in light of transformative experiences and ideas (Paul, 2014).

Keywords: Awareness, unawareness, updating, transformative experiences, reverse Bayesianism, second-order beliefs, hypothesis testing.

JEL-Classifications: D83.

1 Introduction

Reverse Bayesianism has been proposed for comparing beliefs under unawareness of some events with beliefs under awareness of those events. When this comparison takes place in a temporal context, it is interpreted as updating of beliefs upon becoming aware (Karni and Vierø, 2013, 2017, Torres, 2010, Hayashi, 2012, Wenmackers and Romeijn, 2016, Dominiak and Tserenjigmid, 2018, 2022, Dietrich, 2018, Karni, Valenzuela-Stookey, and Vierø, 2021, Vierø, 2021, Dominiak, 2022, Chakravarty, Kelsey, and Teitelbaum, 2022). In an atemporal context, it is interpreted as a consistency condition that allows us to

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focus the analysis on the change of awareness rather than a change of information (Heifetz et al., 2013, Meier and Schipper, 2014).[1]

At the core, upon becoming aware of additional events[2], reverse Bayesianism preserves the relative likelihood ratios for events the agent had been aware of before. However, it does not pin down precisely the posterior.[3] It also seems to put a lot of confidence in the prior in the sense that whatever the agent becomes aware of, it will not shatter the relative likelihood ratios for events that the agent has already been aware of. It is not immediately clear why an agent should adhere to these relative likelihood ratios when her basis for forming the prior, the events captured in the prior space, turns out to be defective and incomplete. We believe that reverse Bayesianism is more plausible when becoming aware of non-transformative events rather than transformative events (see Paul, 2014).

We will show in this note that when the agent lacks even some tiny bit of confidence in her prior, the feature of reverse Bayesianism of not pinning down precisely the posterior may lead to updated beliefs that seemingly do not satisfy reverse Bayesianism. This can happen despite the fact that the second-order belief that in our setting models her (lack of) confidence in the prior, satisfies (reverse) Bayesianism. More precisely, we assume that the agent has a second-order belief about priors. Similar to the model of Ortoleva (2012), we assume that she picks the prior with the highest second-order probability. Upon becoming aware, she updates all priors using reverse Bayesian updating. She also considers a new second-order belief that is a reverse Bayesian update of her earlier second-order belief. Since reverse Bayesianism does not pin down the posterior precisely, we can demonstrate situations in which the posterior selected by the reverse Bayesian second-order belief does not select a reverse Bayesian update of the prior selected by original second-order belief. To an outsider, the agent does not appear to be a reverse Bayesian.

Our observation can be interpreted as a critique of reverse Bayesianism: Reverse Bayesianism may break down when the agent lacks full confidence in her prior. However, our exposition can also be interpreted as a generalization of reverse Bayesianism since in our setting the agent still picks a reverse Bayesian update of a prior, just not the one of the prior she picked before becoming aware. Quite naturally, if the agent lacks confidence in the prior, she might pick a reverse Bayesian update of another prior upon becoming aware because her foundation for picking her original prior has been shattered. While reverse Bayesianism may be appropriate for non-transformative experiences (Paul, 2014)

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[1] This is somewhat analogous to the common prior assumption. The common prior assumption is not necessarily made for realism. Rather, it allows us to theoretically isolate the effect of information received at interim; see for instance Aumann (1974) for a seminal illustration of the role of the common prior assumption and Morris (1995) for a discussion.

[2] For the sake of the exposition, we focus on the temporal interpretation of reverse Bayesianism.

[3] This issue has also been explicitly recognized by Chakravarty, Kelsey, and Teitelbaum (2022), who impose an additional assumption that pins down the reverse Bayesian update.

and ideas[4], our extension of reverse Bayesianism may fit also settings in which the agent has transformative experiences and ideas that also lets her reconsider her prior.

Our approach can be interpreted as extending Karni and Vierø (2013) with ideas of Ortoleva (2012). Ortoleva (2012) (see also Ortoleva, 2022) proposes an approach in which the agent has a second-order belief over first-order priors. She picks the first-order prior that has the highest second-order probability. If an event occurs, she first checks whether this event has at least epsilon probability w.r.t. her picked first-order prior. If yes, she updates according to Bayesian conditioning. Otherwise, she dismisses her picked first-order prior, updates her second-order belief in light of the event, and picks the first-order prior that has now highest updated second-order belief. Our procedure is inspired by this approach but differs in several respects: First, an event does not have to occur for updating the prior. In our case, it is enough that the agent becomes aware of it. That is, she discovers that an event that she has previously not conceived of may or may not occur. In our case, the agent may dismiss a model not just because of new data but also because of new ideas. Second, by definition of unawareness, the agent never assigns any prior probability (not even zero probability) to an event she is unaware of. Thus, when she becomes aware of an event, she always revises her prior as the epsilon-threshold is muted for changes of awareness. Third, upon becoming aware, she considers the reverse Bayesian update of her second-order belief. This is a belief is over reverse Bayesian updates of all her first-order priors. That is, in our approach also the set of priors changes upon becoming aware (while it stays the same in Ortoleva's approach). This is one of the reasons why updating conditional on an event that had zero probability with the picked prior in Ortoleva (2012) is different from updating upon becoming aware in our setting.[5]

We are not the first one discussing the pros and cons of reverse Bayesianism and potential generalizations. In philosophy, reverse Bayesianism has been defended by Wenmackers and Romeijn (2016), Bradley (2017), Roussos (2021), and De Canson (2024), and criticized by Mahtani (2021) and Steele and Steffansson (2021). In decision theory, a critique is presented by Chambers and Hayashi (2018) but closer inspection reveals that their critique is aimed at the particular structure of state-spaces in Karni and Vierø (2013) rather than reverse Bayesianism. Reverse Bayesianism has been generalized by Piermont (2021) and Dominiak and Tserenjigmid (2022), "specialized" with an additional assumption by Chakravarty, Kelsey, and Teitelbaum (2022), and related to prediction

[4]Ma and Schipper (2017) tested a main assumption behind reverse Bayesianism, namely risk preferences invariant to changes of awareness, and were not able to reject it. However, their tests did not involve becoming aware of transformative experiences but just additional outcomes in some lotteries. See Araujo and Piermont (2023) for related experimental results.

[5]Of course, a probability zero event differs already from an event that the agent is unaware of because an agent is unaware of an event if and only if she is unaware of the negation of the event while when an agent assigns zero probability to an event, she is certain of its complement and thus would happily bet her entire wealth on the complement. Schipper (2013) uses this observation to provide a procedure to reveal whether an agent assigns zero probability to an event or is unaware of it. An agent is unaware of an event if and only if she acts as if both the event and its complement are null.

rules for novelty in population biology and combinatorial probability theory by Schipper (2024). A first experimental study of reverse Bayesianism is presented by Becker et al. (2022) although the main assumption underlying reverse Bayesianism, namely risk preferences invariant to changes of awareness, has been previously tested by Ma and Schipper (2017) and Araujo and Piermont (2023). It has been applied to law and economics by Chakravarty, Kelsey and Teitelbaum (2024) and to strategic interaction in information economics by Francetich and Schipper (2024).

We must point out that despite the extant criticisms of reverse Bayesianism and the criticism put forward in this note (if it is interpreted as such), we strongly believe that the heydays of reverse Bayesianism lie still ahead. The reason is simply that when developing more and more applications of awareness to information economics, reverse Bayesianism allows us to compare conditions on relatively likelihoods across awareness levels. For instance, for screening problems and optimal mechanisms, it is important to study the monotonicity of reverse inverse hazard rates (i.e., the Mills' ratio), a condition on relative likelihood ratios. Reverse Bayesianism often helps us to preserve or rank these relative likelihood ratios upon changes of awareness, and thus it helps us to derive results on how optimal menus of contracts or optimal mechanisms change with changes of awareness (see Francetich and Schipper, 2024, for an application to screening).

2 How Reverse Bayesianism Works

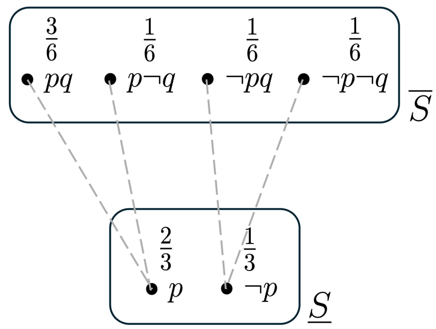
Becoming aware is formalized as an elaboration of the state space. Some of the literature distinguished between two formalizations of such elaborations: a proper elaboration of all states (i.e., also somewhat misleadingly called a refinement of states)[6] and an addition of states (also called expansion of the state space)[7].

Figure 1 illustrates reverse Bayesianism with a proper elaboration of all states. There are two state spaces, the prior state space $\underline{S}$ and the elaborated state space $\overline{S}$. For illustration, we print beside each state propositions that are true at that state. For instance, in the second left-most state of space $\overline{S}$ proposition p is true and proposition $\neg q$ is true. In fact, we can think of states in $\overline{S}$ as describing combinations of truth and falsehoods for propositions p and q while states in $\underline{S}$ describe situations in which only proposition p is true or false. Clearly, $\underline{S}$ is a "poorer" space than $\overline{S}$ in the sense that nothing can be described about proposition q. There is a surjective projection from the "richer" space $\overline{S}$ to the "poorer" space $\underline{S}$ indicated by faint dashed lines ensuring that each state in $\underline{S}$ has elaborations in the space $\overline{S}$. The prior before becoming aware is given by the probability measure $\left(\frac{2}{3}, \frac{1}{3}\right)$ on $\underline{S}$. That is, (the state with) p is twice as likely as (the state with) $\neg p$. A reverse Bayesian update is given by the probability measure

[6] This occurs for instance when the agent discovers additional actions in the particular formalization of state spaces by Karni and Vierø (2013).

[7] In the particular formalization of state spaces by Karni and Vierø (2013), the expansion occurs when the agent becomes aware of additional consequences.

Figure 1: Reverse Bayesianism with Proper Elaboration of All States

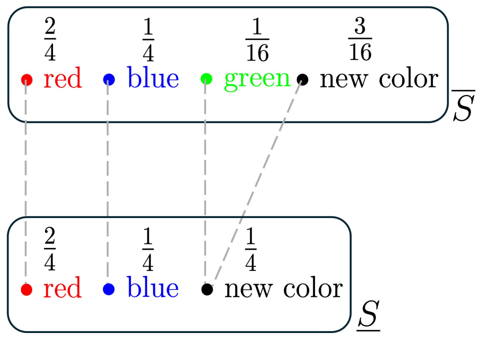


$\left(\frac{3}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$ on $\overline{S}$. That is, pq is three times as likely as any of the other states. We observe Reverse Bayesianism update preserves the relative likelihood ratios for events p and $\neg p$ of the prior. That is, in $\overline{S}$, the event p consisting of the union of states pq and $p\neg q$ is twice as likely as the event $\neg p$ consisting of the union of states $\neg pq$ and $\neg p\neg q$. We also observe that the prior on $\underline{S}$ is the marginal of the Reverse Bayesian update on $\overline{S}$, a feature of reverse Bayesianism when raising awareness comes in form of a proper elaboration of all prior states. Both probability measures form a system of probability measures as in Heifetz et al. (2013).

Figure 2 illustrates reverse Bayesianism with an expansion of the state space by additional states. Again, there are two state spaces, the prior state space $\underline{S}$ and the elaborated state space $\overline{S}$. For illustration, we can think of sampling balls of various colors from an urn with balls of unknown colors (as in the literature of sampling novel species; see Zabell, 1997, Pitman, 1995, and Schipper, 2024). Initially, with space $\underline{S}$, the agent knows that there are red and blue balls, and that the frequency of red balls is $\frac{1}{2}$ while with frequency $\frac{1}{4}$ she encounters a blue ball. Moreover, with probability $\frac{1}{4}$, she expects a ball of a new color different from red or blue. Upon sampling such a new color, let's say a green ball, she now knows that there are red, blue, and green balls in the urn. A reverse Bayesian update is given by $\left(\frac{2}{4}, \frac{1}{4}, \frac{1}{16}, \frac{3}{16}\right)$. Again, we observe that it preserves the relative likelihood of balls of previously known colors (i.e., red and blue).[8] We also indicated the surjective projections from space $\overline{S}$ to space $\underline{S}$. Not surprisingly, red projects exactly to red and blue projects exactly to blue. What is elaborated is the state "new color" as new colors are discovered; see also Wenmackers and Romeijn (2016). Sometimes no awareness of unawareness is considered when discussing expansion of the state space in reverse Bayesianism, i.e., the state "new color" is neglected. However, it does not matter for our discussions as reverse Bayesian only applies to non-null states

[8] This particular example suggests a strengthening of reverse Bayesianism in the context of sampling novelty. Not just relative likelihoods are preserved for red and blue but even their probabilities are preserved. Karni and Vierø (2013) do not feature this stronger property when they consider expansion of the state space upon becoming aware of additional consequences.

Figure 2: Reverse Bayesianism with Expansion



(see Karni and Vierø, 2013). The formalization here emphasizes how it fits with the broader literature on unawareness, in particular models of multiple state spaces (e.g., Modica and Rustichini, 1999, Heifetz et al., 2006, 2008, 2013, Galanis, 2013).

In both cases of reverse Bayesianism, the proper elaboration of states and the expansion of the state space, we like to draw attention to two features of reverse Bayesianism:

1. Reverse Bayesianism does not pin down precisely the posterior. It just requires that the relative likelihoods ratios for previously aware events remain unchanged. There are many posteriors on $\overline{S}$ that would satisfy this property.

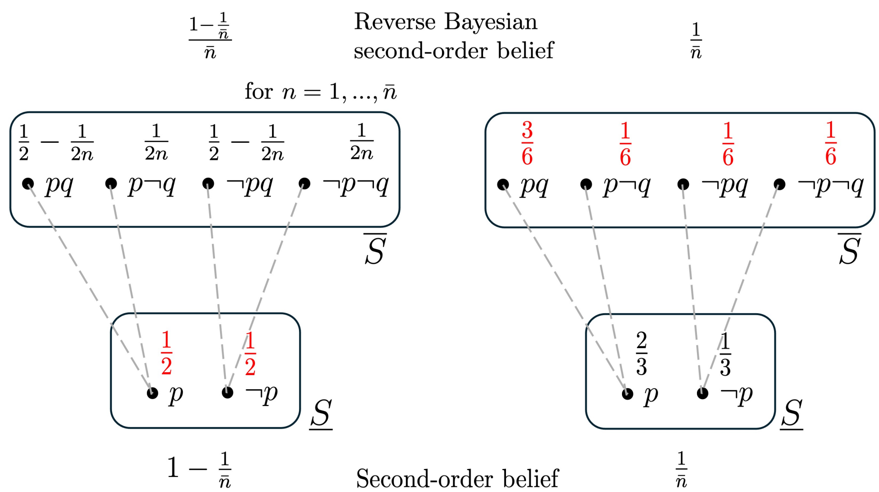
2. Reverse Bayesianism seem to put a lot of confidence in the prior in the sense that whatever the agent becomes aware of will not shatter the relative likelihood ratios for previously aware events. It is not clear why an agent should adhere to these relative likelihood ratios when her basis for forming the prior, the events captured in the prior space $\underline{S}$, turns out to be defective and incomplete.

3 An Example

Consider a setting similar to Figure 1 but assume that agent considers a prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ on $\underline{S}$ as in the left part of Figure 3 that puts equal probability on p and $\neg p$ in $\underline{S}$. However, the agent may lack confidence in the prior. She may alternatively consider the prior $\left(\frac{2}{3}, \frac{1}{3}\right)$ as in the right part of Figure 3 and earlier in Figure 1. Since we want to allow for arbitrary small lack of confidence, it will be convenient to parameterize the second-order beliefs over priors with an integer $\bar{n} \in \mathbb{N}$ with $\bar{n} > 1$. In particular, the agent may assign second-order probability $1 - \frac{1}{\bar{n}}$ to the prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ and probability $\frac{1}{\bar{n}}$ to the prior $\left(\frac{2}{3}, \frac{1}{3}\right)$. The larger $\bar{n}$, the larger is her confidence in her prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ as printed below the lower state spaces in Figure 3.

Since for any $\bar{n} > 2$, the second-order belief assigns a larger probability to the prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ than to the prior $\left(\frac{2}{3}, \frac{1}{3}\right)$, the agent picks prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ as in the decision theory of

Figure 3: The Example



Ortoleva (2012).[9] We indicate this choice in Figure 3 by the red coloring of the prior $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Now consider a class of reverse Bayesian updates of $\left(\frac{1}{2}, \frac{1}{2}\right)$ on $\overline{S}$ conveniently parameterized by $n = 1, ..., \bar{n}$ given by $\left(\frac{1}{2} - \frac{1}{2n}, \frac{1}{2n}, \frac{1}{2} - \frac{1}{2n}, \frac{1}{2n}\right)$. Each of these beliefs are a reverse Bayesian update of $\left(\frac{1}{2}, \frac{1}{2}\right)$ on $\overline{S}$. Here we use fact 1, namely, that reverse Bayesianism does not exactly pin down the posterior. As already in Figure 1, we also a consider a reverse Bayesian update of $\left(\frac{2}{3}, \frac{1}{3}\right)$ on $\overline{S}$ given by $\left(\frac{2}{4}, \frac{1}{4}, \frac{1}{16}, \frac{3}{16}\right)$. That is, in contrast to the collection of reverse Bayesian updates of prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ we just consider a single reverse Bayesian update of the prior $\left(\frac{2}{3}, \frac{1}{3}\right)$. Again, we are free to do so because by fact 1, reverse Bayesianism does not exactly pin down the posterior. We print the reverse Bayesian updates in the upper state spaces of Figure 3.

For the second-order belief update, we will require that it satisfies reverse Bayesianism by assuming that it assigns probability $\frac{1-\frac{1}{\bar{n}}}{\bar{n}}$ to $\left(\frac{1}{2} - \frac{1}{2n}, \frac{1}{2n}, \frac{1}{2} - \frac{1}{2n}, \frac{1}{2n}\right)$ for each $n = 1, ..., \bar{n}$ and probability $\frac{1}{\bar{n}}$ to $\left(\frac{2}{4}, \frac{1}{4}, \frac{1}{16}, \frac{3}{16}\right)$. Since there are $\bar{n}$ beliefs of the form $\left(\frac{1}{2} - \frac{1}{2n}, \frac{1}{2n}, \frac{1}{2} - \frac{1}{2n}, \frac{1}{2n}\right)$ (for $n = 1, ..., \bar{n}$), all reverse Bayesian updates of $\left(\frac{1}{2}, \frac{1}{2}\right)$ together have weight $1 - \frac{1}{\bar{n}}$ in the reverse Bayesian second-order belief, the same as the second-order belief assigns to the prior $\left(\frac{1}{2}, \frac{1}{2}\right)$. And since there is just one reverse Bayesian update of $\left(\frac{2}{3}, \frac{1}{3}\right)$, it is assigned weight $\frac{1}{\bar{n}}$ by the reverse Bayesian second-order belief, the same as with the second-order belief.

[9] Allowing for doubt about priors via second-order beliefs is not just common in Bayesian learning (e.g., Bayesian learning in multi-armed bandits) but also theories of ambiguity (see Klibanoff, Marinacci, and Mukerji, 2005, Nau, 2006, Seo, 2009, and Ergin and Gul, 2009). Latter features an agent's unwillingness or inability to compound (with the same attitude towards first and second-order uncertainty). Ortoleva's (2012) approach can be seen as an extreme form of an unwillingness to compound as familiar from hypothesis testing widely used in empirical sciences (as opposed to methods of Bayesian statistics).

Again, similar to the decision theory of Ortoleva (2012), we assume that the agent picks the posterior with the highest (reverse Bayesian) second-order probability. Note though that $\frac{1-\frac{1}{\bar{n}}}{\bar{n}} < \frac{1}{\bar{n}}$ for $\bar{n} \geq 1$. Thus, upon becoming aware, the agent's belief is $\left(\frac{2}{4}, \frac{1}{4}, \frac{1}{16}, \frac{3}{16}\right)$, which is *not* a reverse Bayesian update of her prior $\left(\frac{1}{2}, \frac{1}{2}\right)$ selected by the second-order belief. Note that by increasing $\bar{n}$ we can make the agent's doubt over her prior arbitrarily small. Still the agent would appear to violate reverse Bayesianism in this example.

4 A Generalization of Reverse Bayesianism

We can develop our observation into a generalization of reverse Bayesianism. Consider two measurable spaces, the poorer space $\underline{S}$ and the richer space $\overline{S}$, with a measurable surjective projection from $\overline{S}$ to $\underline{S}$ denoted by $r : \bar{S} \longrightarrow \underline{S}$. When state $\bar{s} \in \bar{S}$ occurs for an agent who can describe events in $\bar{S}$, then $r(\bar{s})$ occurs for any one who can describe only events in $\underline{S}$.[10] The set of priors is $\Delta(\underline{S})$. Rather than assuming that the agent has exactly one prior, we allow for some lack of confidence. We assume that she has a second-order belief over priors, $\underline{\sigma} \in \Delta(\Delta(\underline{S}))$. For instance, when modeling arbitrary small lack of confidence in the prior, $\underline{\sigma}$ may assign $1 - \varepsilon$ to one prior $\underline{\mu}^* \in \Delta(\underline{S})$ and a small $\varepsilon > 0$ to some priors in $\Delta(\underline{S}) \setminus \{\underline{\mu}^*\}$. As in Ortoleva (2012), we assume that the agent selects her most probable prior. I.e., her initial prior satisfies $\{\underline{\mu}^*\} = \arg\max_{\underline{\mu} \in \Delta(\underline{S})} \underline{\sigma}(\underline{\mu})$. We say that her second-order belief $\underline{\sigma}$ selects the first-order belief $\underline{\mu}^*$.

Upon becoming aware of $\bar{S}$, she considers beliefs in $\Delta(\bar{S})$. Note that any $\bar{\mu} \in \Delta(\bar{S})$ is a reverse Bayesian update of some $\underline{\mu} \in \Delta(\underline{S})$. I.e., $\bar{\mu}$ is a reverse Bayesian update of $\underline{\mu}$ if $\underline{\mu} = \bar{\mu}_{\underline{S}} = marg_{\underline{S}}\ \bar{\mu}$ where $\bar{\mu}_{\underline{S}} = marg_{\underline{S}}\ \bar{\mu}$ is defined by for any $E \subseteq \underline{S}$, $\bar{\mu}_{\underline{S}}(E) := \overline{\mu}(r^{-1}(E))$.

We consider both $\Delta(\underline{S})$ and $\Delta(\bar{S})$ as measurable spaces.[11] We define a measurable surjective projection $\rho : \Delta(\bar{S}) \longrightarrow \Delta(\underline{S})$ by $\rho(\mu) = \mu_{\underline{S}}$ for any $\mu \in \Delta(\bar{S})$. That is, μ projects to $\mu_{\underline{S}}$ if $\mu_{\underline{S}}$ is the marginal of μ (or μ is a reverse Bayesian update of $\mu_{\underline{S}}$.)

Let $\bar{\sigma}$ be a reverse Bayesian update of $\underline{\sigma} = \bar{\sigma}_{\underline{S}} = marg_{\Delta(\underline{S})}\ \bar{\sigma}$ defined by for any measurable subsets of priors $M \subseteq \Delta(\underline{S})$, $\bar{\sigma}_{\underline{S}}(M) = \bar{\sigma}(\rho^{-1}(M))$.

The example of Section 3 raises the question of when does the reverse Bayesian update of the second-order belief select a reverse Bayesian update of the first-order belief? I.e., when is it the case that $\{\mu\} = \arg\max_{\mu' \in \Delta(\bar{S})} \sigma(\mu')$ with $\{\mu_{\underline{S}}\} = \arg\max_{\mu'' \in \Delta(\underline{S})} \sigma_{\underline{S}}(\mu'')$? We can find counterexamples like the one above where the second-order belief $\sigma \in \Delta(\Delta(\bar{S}))$ is "more random" on the Bayesian updates of $\mu_{\underline{S}}$ selected by $\sigma_{\underline{S}}$ than on the reverse Bayesian updates of priors different from $\mu_{\underline{S}}$. Since reverse Bayesianism does not

[10]Obviously, this a special case of a complete lattice of spaces in Heifetz et al. (2006, 2013). The focus on this special case will suffice for our exposition.

[11]For the sake of concreteness, we can endow $\Delta(\underline{S})$ with a sigma-field generated by sets of the form $\{\mu \in \Delta(\underline{S}) \mid \mu(E) \geq p\}$ for any measurable set E of $\underline{S}$ and $p \in [0, 1]$.

pin down precisely the posterior, there are many possible reverse Bayesian posteriors, which allows for such second-order uncertainty over posteriors. So the feature of reverse Bayesianism of not pinning down precisely the posterior is necessary for such a counterexample. Having some doubt about the prior is also necessary for such a counterexample since otherwise any reverse Bayesian second-order belief must assign all probability only to reverse Bayesian updates of the selected prior.

However, the setting also suggests a generalization of reverse Bayesianism:

1. At awareness level $\underline{S}$, consider priors in $\Delta(\underline{S})$ and form a second-order belief over priors, $\underline{\sigma} \in \Delta(\Delta(\underline{S}))$.

2. Pick a prior in $\Delta(\underline{S})$ with the largest second-order probability, $\underline{\mu}^* \in \arg\max_{\underline{\mu} \in \Delta(\underline{S})} \underline{\sigma}(\underline{\mu})$.

3. Upon becoming aware, at awareness level $\overline{S}$, consider priors in $\Delta(\overline{S})$ according to a reverse Bayesian second-order belief $\overline{\sigma} \in \Delta(\Delta(\overline{S})$, i.e., a belief $\overline{\sigma}$ such that for any measurable subsets of priors $M \subseteq \Delta(\underline{S})$, $\overline{\sigma}(\rho^{-1}(M)) = \underline{\sigma}(M)$.

4. Pick a prior in $\Delta(\overline{S})$ with the largest second-order probability, $\overline{\mu}^* \in \arg\max_{\overline{\mu} \in \Delta(\overline{S})} \overline{\sigma}(\overline{\mu})$. This is a "generalized" reverse Bayesian update.

In the special case, when $\underline{\sigma}$ is a Dirac measure assigning probability one to a particular prior $\underline{\mu}^* \in \Delta(\underline{S})$, the procedure amounts to "standard" reverse Bayesianism. Otherwise, the example shows that the procedure may pick a reverse Bayesian update of a different prior. This pick is guided by a reverse Bayesian second-order belief.

Note that with "generalized" reverse Bayesianism, the decision make does not necessarily appear to be a reverse Bayesian to an outside observe even though she is in some sense a "hyper-reverse" Bayesian because she also applies reverse Bayesianism to second-order beliefs. In any case, the idea inspired by Ortoleva (2012) of picking the most likely prior from a set of priors and the most likely posterior from a set of posteriors adds a non-Bayesian flavor to the generalization of reverse Bayesianism.

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