Raising Bidders' Awareness in Second-Price Auctions*

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Abstract

When bidders bid on complex objects, they might be unaware of characteristics effecting their valuations. We assume that each buyer's valuation is a sum of independent random variables, one for each characteristic. When a bidder is unaware of a characteristic, he omits the random variable from the sum. We study the seller's decision to raise bidders' awareness of characteristics before a second-price auction with entry fees. Optimal entry fees capture an additional unawareness rent due to unaware bidders misperceiving their probability of winning and the price to be paid upon winning. When raising a bidder's individual awareness of a characteristic with positive expected value, the seller faces a trade-off between positive effects on the expected first order statistic and unawareness rents of remaining unaware bidders on one hand and the loss of the unawareness rent from the newly aware bidder on the other. We present characterization results on raising public awareness together with no versus full information. We discuss the winner's curse due to unawareness of characteristics.

Keywords: Unawareness, disclosure, optimal second-price auctions with independent private values, winner's curse, entry fees.

JEL-Classification: C72, D44, D83.

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1   Introduction

We study raising bidder's awareness before second-price auctions with independent private values. An auctioneer wants to sell an object to one of n risk-neutral bidders. The bidders' valuations for the object depend on multiple characteristics, some of which bidders might be even unaware of. That is, a bidder may not conceive of all characteristics of the object that may affect his valuation. Such situations are common when the object is for instance an "experience good" whose characteristics become only transparent when it is used by the future winning bidder. In such contexts, it is also natural to assume that the auctioneer, as a prior owner, has more experience with the object and is already aware of the relevant characteristics. Consequently, the auctioneer may now decide to strategically raise the bidders' awareness of some characteristics but not of others, and may also commit to disclose some amount of information on these characteristics. We emphasize that raising awareness of a characteristic just means telling the bidder about the existence of the characteristic. This is different from providing information about the value of the characteristic. Latter implies former but not vice versa. To emphasize the difference between awareness and information, consider a statement like "the neighbor of the property may or may not have a prescriptive easement". Such a statement surely raises awareness of the characteristic "prescriptive easement" but does not provide information about the likelihood of such an easement because the statement is a tautology, i.e., an event that always obtains. Similarly, a statement such as "the neighbor has a prescriptive easement and does not have a prescriptive easement" raises awareness of such an easement in the sense of letting the recipient of the statement think about it but it does not provide information about it because it is a contradiction, i.e., an event that never obtains. Without awareness of a characteristic, the bidder cannot form beliefs about the value of the characteristic. In contrast, disclosure of information means that also some information is provided about the value of the characteristic like for instance "it is more likely than not that the neighbor has a prescriptive easement on the property". Such a statement not only raises awareness of a prescriptive easement but also provides information on it.

Our work is inspired by Szech (2011). She studied the (possibly asymmetric) disclosure of costly information before second-price auctions with entry fees in a setting of independent private values and risk neutral bidders. We generalize her characterization of optimal entry fees (her Proposition 1) to settings that allow the seller to potentially raise asymmetric awareness (see our Proposition 1 and Corollary 1). Yet, while she then studies the surprising optimality of asymmetric disclosure of costly information to bidders, we focus on costless raising awareness and costless disclosure of information.

Before describing our approach and results as well as the connection to the rest of

the literature, we like to mention as a motivation three potential applications:

Firms competing in an auction to take over a firm: The owner of the firm, as an insider, has not just better information on the firm but is also aware of all hidden details of the business of the firm including potential law suits, innovative products, potential accounting problems, etc. while some of the bidders may not even think about them. The owner has to decide of what she raises the bidders' awareness and how much information she provides beyond mandatory disclosure requirements. A similar motivation, but just w.r.t. information, has been used by Bergemann and Pesendorfer (2002), Ganuza (2004), Eso and Szentes (2007), and Szech (2011) among others, and seems to be based on Burkart (1995).

Bidding for access to internet users: Advertisers compete in an auction for displaying advertisement to users of websites, search engines, and apps. Internet firms such as Google, Facebook etc. collect massive amounts of data on individual users. Big data about users allow for inferences of detailed psycho-demographic profiles of internet users (Kosinski, Stillwell, and Graepel, 2013). These profiles are of potential interest to advertisers for estimating their willingness to pay for an ad shown to the user. Yet, advertisers may not be aware of all dimensions on which data is collected and some of the information may not be provided to advertisers either for strategic reasons or because of privacy laws. The internet firms still can decide about what to tell the advertisers about the characteristics on which data is collected on, even if the data cannot be provided to advertisers.[1]

Procurement/Sale for complex projects/commodities: Potential contractors compete for an award of a complex project/commodity such as for a novel weapon technology, a power plant, a property etc. These projects involve many dimensions relevant to quality, costs, values etc. Contractors may not be aware of some of those dimensions as they are not necessarily transparent to outsiders. In contrast, the project owner may be aware of those dimensions and may have information on those dimensions. Before running the procurement or sales auction, the owner can decide about the issues she wants to make bidders aware and commit on the information she provides on these issues.

To model unawareness of characteristics relevant to the bidders' valuations, we allow in Section 2 for multi-dimensional valuations in form of random vectors with each component of the vector representing the value of one characteristic. A bidder's valuation is now the sum of the component random variables. Yet, the bidder may be unaware of some of those dimensions, only summing over the random variables of which she is aware. Being unaware of a characteristic and the random variable representing this characteristic also means that the bidder does not consider that other bidders may be aware of it. Even though we use the weak dominant strategy equilibrium of independent private-

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[1]In a recent paper, Gao (2024) studies disclosure of information via big data sets (not in an auction though). She considers selective disclosure of information only, which we can think of represented by the "rows" of a data set (i.e., the individual observations). In contrast, we think of raising awareness of "columns" of a data set, the characteristics on which data is collected.

value second-price auctions, these beliefs about other's awareness are relevant because we are interested in optimal entry fees that the auctioneer can charge, which depend on the bidders' expected profit. We model payoff type spaces with unawareness as a restricted version of unawareness type spaces by Heifetz, Meier, and Schipper (2013a). Our model comprises of a finite lattice of payoff type spaces with projections, one payoff type space for every subset of characteristics (that include a "default" characteristic) modeling differences in awareness. The lattice order is naturally induced by set inclusion on the subsets of characteristics. Information is modeled via sigma algebras on those payoff type spaces. When the seller discloses a sigma algebra to a bidder, the payoff type space on which the sigma algebra is defined represents the awareness level and the elements of the sigma algebra the information.

Our first result in Section 3, Proposition 1, characterizes the auctioneer's optimal entry fees and expected revenue from the weak dominant strategy equilibrium of the second-price auction. Compared to Szech (2011), we obtain an additional term in the auctioneer's expected revenue that for each bidder captures the difference of the bidder's expected profit and the bidder's expected profit from the perspective from someone with full awareness. This unawareness rent is only present when the bidder is unaware of a characteristic and bidders have asymmetric awareness. It is caused by the bidder's misperception of his probability of winning and his misperception of the expected price he has to pay upon winning.

Armed with the characterization of optimal entry fees given awareness and information, we study in Section 4 how to optimally raise individual bidder's awareness. We show that if a characteristic has positive expected value, then the auctioneer wants to make at least one bidder aware of it, creating an asymmetry in awareness and thus unawareness rents in addition to increasing the expected (largest) first order statistic. Raising further bidders' awareness involves now a trade-off: It increases further the expected first order statistic. It also increases further the unawareness rents accrued from remaining unaware bidders (because they overestimate their probability of winning and underestimate the expected price they have to pay upon winning even more). However, the auctioneer also loses now the unawareness rent from the bidder whom she made aware of the characteristic. We show by examples that a characteristic having positive expected value is not necessary for raising awareness of the characteristic.

We also consider optimally raising public awareness, when any awareness disclosed must be disclosed to all bidders. We show that the auctioneer raises public awareness of a characteristic with no information if and only if the expected value of the characteristic is positive. When raising awareness involves mandatory disclosure of full information, the auctioneer keeps the public unaware of a characteristic if and only if the characteristic contributes negatively to the first order statistic. This also implies that the expected value of the characteristic must be negative.

In Section 6.1, we discuss the winner's curse due to unawareness of characteristics. When a seller optimally raises awareness, then any characteristics of which bidders are

kept unaware must have negative expected value. Thus, unaware winners pay more in expectation than what they would if they were aware of all characteristics. This winner's curse due to unawareness occurs even with independent private values and is therefore different from the winner's curse in common value auctions.

In Section 6.2, we discuss how awareness differs from information. We show that given common awareness of a set of characteristics, the auctioneer wants to optimally disclose all information on these characteristics. This generalizes an observation by Szech (2011), who noted that in the absence of information costs the auctioneer wants to disclose all information because of the convexity of the expected maximum. It is in contrast to Ganuza (2004), who finds that the auctioneer does not always want to disclose all information before second-price auctions. However, he does not allow for entry fees (see also Board, 2009, Hagedorn, 2009, Bergemann et al., 2022). Our result is reminiscent of Eso and Szentes (2007) who also show the optimally of full disclosure but in a more general mechanism design setting (see also Gershkov, 2009).[2]

A discussion of the related literature on contracting under unawareness is contained in Section 6.3. Further mathematical details that should be useful in future applications are collected in Appendix A. Proofs are relegated to Appendix B.

2   Varying Dimensions of Characteristics

Consider a seller who wants to sell one indivisible object to one of $N = \{1, ..., n\}$ risk-neutral buyers via a second-price auction with entry fees. Each buyer's willingness to pay for the object depends on the values of characteristics of the object. There are up to $M = \{1, ..., m\}$ characteristics. The values of characteristics are coded into a m-dimensional real-valued vector of variables. That is, each characteristic is represented by one dimension of the vector.

Bidders do not know the value of each characteristics nor are they necessarily aware of all characteristics. Before the auction, the seller can decide for each bidder $i \in N$ of which characteristics $M^i \subseteq M$ she wants to make him aware. Moreover, she can also decide on how much information the bidder can learn about the value of these characteristics. This requires us to consider second-price auctions with multi-dimensional valuations in which the dimension may vary with the disclosure decision by the seller.

Let S_M be a measurable space of states with a sigma-algebra $\mathcal{F}_M$. Define $\mathcal{M} = \{M' \in 2^M : 1 \in M'\}$. This is the set of all subsets of characteristics that contain the first characteristic. Since M is finite, we have that $\mathcal{M}$ is finite. The reason for considering only subsets of characteristics that contain the first characteristic is that we want valuations to be always well-defined. That is, all bidders are always aware of the

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[2]Other papers on information disclosure before auctions include Milgrom and Weber (1982), Rasmusen (2006), Bergemann and Pesendorfer (2007), Vagstad (2009), Ganuza and Penalva (2010, 2019), and Arefeva and Meng (2021).

first characteristic. For each $M' \in \mathcal{M}$, we let $S_{M'}$ be a measurable space of states with sigma-algebra $\mathcal{F}_{M'}$. For any $M', M'' \in \mathcal{M}$, $M' \neq M''$, we let $S_{M'}$ and $S_{M''}$ be disjoint. The set of all such spaces is denoted by $\mathcal{S} = \{S_{M'}\}_{M' \in \mathcal{M}}$. Observe that $\mathcal{S}$ is a finite lattice of disjoint spaces with the lattice order induced by set inclusion of subsets in $\mathcal{M}$ with the unique greatest element S_M and the unique least element $S_{\{1\}}$. We let $\Omega := \bigcup_{M' \in \mathcal{M}} S_{M'}$ be the (disjoint) union of spaces. For every $M'', M' \in \mathcal{M}$ with $M' \subseteq M''$, there is a measurable surjective projection $r_{M'}^{M''} : S_{M''} \longrightarrow S_{M'}$ that satisfies (i) $r_{M'}^{M'''} = r_{M'}^{M''} \circ r_{M''}^{M'''}$ for every $M''', M'', M' \in \mathcal{M}$ with $M' \subseteq M'' \subseteq M'''$, and (ii) $r_{M'}^{M'} = id_{M'}$ (i.e., the identity on $S_{M'}$) for every $M' \in \mathcal{M}$. These projections allow us to model events across spaces. The lattice structure is a special case of unawareness structures by Heifetz, Meier, and Schipper (2013a). It enables us to model situations in which the seller keeps some bidders unaware of some characteristics of the object.

For every characteristic $j \in M$ and and bidder $i \in N$, we introduce a measurable random variable $X_j^i : S_{\{1,j\}} \longrightarrow \mathbb{R}$. We assume that X_j^i is almost surely not constant on $S_{\{1,j\}}$ and integrable. We extend X_j^i to states in spaces in $\{S_{M'} \in \mathcal{S} : j \in M', M' \in \mathcal{M}\}$ by letting for $M' \in \mathcal{M}$ with $j \in M'$ and $\omega \in S_{\{1,j\}}$, $X_j^i(\omega') = X_j^i(\omega)$ for all $\omega' \in (r_{\{1,j\}}^{M'})^{-1}(\omega)$. That is, the random variable on $S_{M'}$ is extended to more expressive spaces using the inverse image with respect to the measurable surjective projections. For every $M' \in \mathcal{M}$, we introduce a measurable random vectors $X_{M'} : \bigcup_{S \in \{S_{M''} \in \mathcal{S} : M' \subseteq M'', M'' \in \mathcal{M}\}} S \longrightarrow \mathbb{R}^{n \cdot |M'|}$ by $X_{M'}(\omega) = (X_j^i(\omega))_{j \in M', i \in N}$ for all $\omega \in \bigcup_{S \in \{S_{M''} \in \mathcal{S} : M' \subseteq M'', M'' \in \mathcal{M}\}} S$. The following lemma allows agents with awareness higher than M' to reason about random vectors $X_{M'}$.

Lemma 1 *For every $M', M'' \in \mathcal{M}$ with $M' \subseteq M''$, $X_{M'}$ is measurable on $S_{M''}$.*

For any $M' \in \mathcal{M}$, let $P^{M'} \in \Delta(S_{M'})$ denote a probability measure on $S_{M'}$. For any $M'', M' \in \mathcal{M}$ with $M' \subseteq M''$, the marginal of $P^{M''}$ on $S_{M'}$, denoted by $P_{|M'}^{M''}$, is defined by $P_{|M'}^{M''}(D) := P^{M''}\left((r_{M'}^{M''})^{-1}(D)\right)$ for $D \in \mathcal{F}_{M'}$. Let $P = (P^{M'})_{M' \in \mathcal{M}} \in \times_{M' \in \mathcal{M}} \Delta(S_{M'})$ be a projective system of probability measures. I.e., $P^{M'}$ is the marginal of $P^{M''}$, for every $M', M'' \in \mathcal{M}$ with $M' \subseteq M''$.

Because $X_{M'}$ is measurable, for any $M' \in \mathcal{M}$, we have by Lemma 1 that for any $x_{M'} \in \mathbb{R}^{n \cdot |M'|}$, that the set $\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\} \cap S_{M''} \in \mathcal{F}_{M''}$ for any $M'' \in \mathcal{M}$ with $M' \subseteq M''$. We define

$$P^{M''}\left(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}\right) := P^{M''}\left(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\} \cap S_{M''}\right)$$

if $M'' \in \mathcal{M}$ such that $M' \subseteq M''$ and let it be undefined otherwise.

The following lemma shows that lattice of measurable spaces with the projective system of probability measures captures the intuition analogous to a standard probability space.

Lemma 2 *For any $M', M'', M''' \in \mathcal{M}$ with $M' \subseteq M'' \cap M'''$ and any $x_{M'} \in \mathbb{R}^{n \cdot |M'|}$,*

$$P^{M''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}) = P^{M'''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}).$$

In light of Lemma 2, we abuse notation and write $P(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\})$ for $P^{M''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\})$ for any $M'' \in \mathcal{M}$ with $M' \subseteq M''$. Moreover, we can now define the distribution function of the random vector X_M by

$$F(x_M) = P(\{\omega \in \Omega : X_M(\omega) \leq x_M\}).$$

Similarly, for any subset of characteristics $M' \in \mathcal{M}$ and nonempty subset of bidders $N' \subseteq N$, we let $F_{M'}^{N'}(x_{M'}^{N'})$ be the marginal distribution, where $x_{M'}^{N'} = (x_j^i)_{j \in M', i \in N'} \in \mathbb{R}^{|N'| \cdot |M'|}$. If no confusion arises, we abuse notation and write $F(x_{M'}^{N'})$ for the marginal distribution.

We adopt the common simplifying assumption in the literature on standard auctions that values are drawn independently across bidders. We will also assume that values of characteristics are drawn independently. The set of random variables $\{X_j^i\}_{j \in M, i \in N}$ is mutually independent. For every $M' \in \mathcal{M}$, every vector $(x_j^i)_{j \in M', i \in N} \in \mathbb{R}^{n \cdot |M'|}$, and every nonempty $N_j \subseteq N$ (where N_j may be interpreted as the subset of bidders who are aware of characteristic j),

$$P\left(\bigcap_{j \in M'} \bigcap_{i \in N_j} \{\omega \in \Omega : X_j^i(\omega) \leq x_j^i\}\right) = \prod_{j \in M'} \prod_{i \in N_j} P\left(\{\omega \in \Omega : X_j^i(\omega) \leq x_j^i\}\right).$$

In terms of distributions, we have that for any $M' \in \mathcal{M}$, $j \in M'$, $\emptyset \neq N_j \subseteq N$, and $x_j^{N_j} = (x_j^i)_{i \in N_j}$,

$$F((x_j^{N_j})_{j \in M'}) = \prod_{j \in M'} \prod_{i \in N_j} F_j^i(x_j^i).$$

We assume that individual marginal distributions for a characteristic are commonly known among all bidders who are aware of the characteristic.

3 Optimal Entry Fees given Disclosure of Awareness and Information

Recall that bidders may not be aware of all characteristics. Initially, they are all aware of characteristic 1 only. Before the auction, the seller may make bidders aware of further characteristics and allow bidders to learn some information on the value of these characteristics. In return, she may charge each bidder an individual participation fee and commits to exclude any bidder from the auction who does not pay the participation

fee. While we assume that the seller is aware of all characteristics, the seller does not know the precise information that the bidder learns after he is charged the participation fee. Formally, the seller describes to each bidder i a sigma-algebra $\mathcal{I}^i_{M'} \subseteq \mathcal{F}_{M'}$ for characteristics $M' \in \mathcal{M}$ on which bidder i will receive information. Note that the seller's description of this sigma-algebra makes the bidder already aware of characteristics in M'. If $M' \in \mathcal{M}$ is the set of characteristics of which bidder i is made aware of by the seller, then we denote $M^i = M'$. We call M^i bidder i's awareness level. The process of information disclosure is inspired by Szech (2011) except that in her model all bidders are aware of all characteristics upfront.

Any bidder may form beliefs about the awareness of other bidders including bidders whom the seller made aware of larger or incomparable subsets of characteristics. We will assume that bidder i forms correct beliefs about the awareness of other bidders subject to his own awareness. That is, bidder i cannot believe that another bidder is aware of a specific characteristic that he himself is unaware of. Formally, we assume that if $(\mathcal{I}^i_{M^i})_{i \in N}$ is the profile of sigma-algebras provided by the seller to bidders, bidder i believes that bidder k is aware of characteristics in $M^i \cap M^k$. That is, $(\mathcal{I}^k_{M^i \cap M^k})_{k \in N}$ is the profile of sigma-algebras perceived by player i, where $\mathcal{I}^k_{M^i \cap M^k}$ is generated by the sets $r^{M^k}_{M^i \cap M^k}(D)$ for $D \in \mathcal{I}^k_{M^k}$. Note that since $1 \in M^i \in \mathcal{M}$ for all $i \in N$, $M^i \cap M^k \neq \emptyset$ for all $i, k \in N$.

We assume for simplicity that for each bidder i, the value of the object is additive in values of characteristics. That is, for the set of characteristics $M' \in \mathcal{M}$, if $(x^i_j)_{j \in M'}$ is the vector of realized values on characteristics $j \in M'$, then $y^i = \sum_{j \in M'} x^i_j$ is bidder i's value of the object. Our approach here is reminiscent of "pseudotypes" in scoring auctions; see, for instance, Asker and Cantillon (2006) or Bajari, Houghton, and Tadelis (2014).

For each bidder $i \in N$ and each subset of characteristics $M^i \in \mathcal{M}$ provided to bidder i, define a random variable $Y^i_{M^i} : \Omega \longrightarrow \mathbb{R}$ by if $\omega \in S_{M'}$ then $Y^i_{M^i}(\omega) = \sum_{j \in M^i \cap M'} X^i_j(\omega)$. To see that this definition makes sense, note first that for any $\omega \in S_{M^i}$, $Y^i_{M^i}(\omega) = \sum_{j \in M^i} X^i_j(\omega)$. Second, for any $\omega \in S_{M'}$ with $M^i \subseteq M'$, we also have $Y^i_{M^i}(\omega) = \sum_{j \in M^i} X^i_j(\omega)$. This may be interpreted as follows. Consider a player who is aware of characteristics in M'. Such a player knows that bidder i is aware only of characteristics in $M^i \subseteq M'$. Hence, she anticipates that bidder i's value is given by $\sum_{j \in M^i} X^i_j(\omega)$. Third, for any $\omega \in S_{M'}$ with $M^i \supseteq M'$, we have $Y^i_{M^i}(\omega) = \sum_{j \in M'} X^i_j(\omega)$. We can interpret this by considering a player who is aware of characteristics in $M' \subseteq M^i$. Such a player anticipates that bidder i is aware of characteristics in M' but since such a player is unaware of characteristics in $M^i \setminus M'$, she cannot anticipate that bidder i is actually aware of $M^i \supseteq M'$. Hence, she views bidder i's perceived value as given by $\sum_{j \in M'} X^i_j \left((r^{M^i}_{M'})^{-1}(\omega) \right)$ which by the definition of X^i_j, $j \in M'$, is $\sum_{j \in M'} X^i_j(\omega)$. Finally, for any $\omega \in S_{M'}$ such that $M' \not\subseteq M^i$ and $M' \not\supseteq M^i$, $Y^i_{M^i}(\omega) = \sum_{j \in M^i \cap M'} X^i_j(\omega)$. This sum is well-defined because $1 \in M' \cap M^i$. To interpret this consider a player who is aware only of characteristics in M'. She must be unaware of characteristics in $M^i \setminus M'$. At the same time, she knows that bidder i is unaware of characteristics in $M' \setminus M^i$.

Thus, she envisions bidder i's value to be $\sum_{j \in M^i \cap M'} X^i_j(\omega)$. To sum up, we define the random variable $Y^i_{M^i}$ such that for any player with awareness M', bidder i's total value is consistent with bidder i's awareness and with M'.

Next, we define conditional expectations of values. For the profile of sigma-algebras $(\mathcal{I}^i_{M^i})_{i \in N}$ that the seller provides to bidders, an agent with awareness level M' calculates for every bidder $i \in N$ an estimated valuation $(\tilde{Y}^i_{M^i})^{M'}$ for the object based on the profile of sigma-algebras he perceives, $(\mathcal{I}^i_{M^i \cap M'})_{i \in N}$. Let $(\mathcal{I}^i_{M^i \cap M'})^{M'}$ be generated by sets $(r^{M'}_{M^i \cap M'})^{-1}(D)$ for $D \in \mathcal{I}^i_{M^i \cap M'}$. Although an agent with awareness level M' considers $\mathcal{I}^i_{M^i \cap M'}$ to be the information received by bidder i, the agent forms belief about states in $S_{M'}$. Thus, we need to consider the set of inverse images of elements in $\mathcal{I}^i_{M^i \cap M'}$ in the space $S_{M'}$. This is the set $(\mathcal{I}^i_{M^i \cap M'})^{M'}$. Given $(\mathcal{I}^i_{M^i \cap M'})^{M'}$, the estimated value of bidder i as perceived by an agent with awareness level M' is the random variable $(\tilde{Y}^i_{M^i})^{M'}$ given by the conditional expectation $\mathbb{E}[Y^i_{M^i} \mid (\mathcal{I}^i_{M^i \cap M'})^{M'}]$ defined for all $D \in (\mathcal{I}^i_{M^i \cap M'})^{M'}$ by

$$
\int_D \mathbb{E}[Y^i_{M^i} \mid (\mathcal{I}^i_{M^i \cap M'})^{M'}](\omega) dP(\omega) \;\; := \;\; \int_D \left(\sum_{j \in M^i \cap M'} X^i_j(\omega) \right) dP(\omega)
$$

$$
= \;\; \sum_{j \in M^i \cap M'} \int_D X^i_j(\omega) dP(\omega),
$$

where the last equation follows from the linearity of the expectations operator.

For the random vector of estimated valuations $((\tilde{Y}^1_{M^1})^{M'}, (\tilde{Y}^2_{M^2})^{M'}, ..., (\tilde{Y}^n_{M^n})^{M'})$ on $(S_{M'}, \mathcal{F}_{M'}, P)$ for $M' \in \mathcal{M}$, consider the order statistics from the point of view of an agent with awareness M' and information $((\mathcal{I}^i_{M^i \cap M'})^{M'})_{i \in N}$, $((\tilde{Y}^{(1)})^{M'}, (\tilde{Y}^{(2)})^{M'}, ..., (\tilde{Y}^{(n)})^{M'})$, with $(\tilde{Y}^{(1)})^{M'}$ being the first order statistic given $((\mathcal{I}^i_{M^i \cap M'})^{M'})_{i \in N}$, $(\tilde{Y}^{(2)})^{M'}$ being the second order statistic given $((\mathcal{I}^i_{M^i \cap M'})^{M'})_{i \in N}$ etc.

Note that the order statistics are defined relative to a set of characteristics $M' \in \mathcal{M}$. That is, order statistics may differ by awareness levels.

Given $((\mathcal{I}^i_{M^i \cap M'})^{M'})_{i \in N}$, let $(\mathbf{1}^i)^{M'}$ denote the variable indicating that bidder i wins from an agent's point of view with awareness M' in the weak dominant strategy equilibrium of the second-price auction. This random variable $(\mathbf{1}^i)^{M'} : S_{M'} \longrightarrow \{0, 1\}$ is defined by

$$
(\mathbf{1}^i)^{M'}(\omega) \;\; = \;\; \begin{cases} 1 & \text{if } (\tilde{Y}^k_{M^k})^{M'}(\omega) \leq (\tilde{Y}^i_{M^i})^{M'}(\omega) \text{ for all } k \neq i, \\ 0 & \text{otherwise.} \end{cases}
$$

If the seller commits to release the information $(\mathcal{I}^1_{M^1}, \mathcal{I}^2_{M^2}, ..., \mathcal{I}^n_{M^n})$, then bidder i's (ex ante) expected probability of winning from the point of view of an agent with awareness

level M' is

$$
\begin{aligned}
\mathbb{E}[(\mathbf{1}^i)^{M'}] &= \int_{S_{M'}} (\mathbf{1}^i)^{M'}(\omega) dP(\omega) \\
&= \int_{\bigcap_{k \neq i}\left\{\omega \in S_{M'}:(\tilde{Y}^k_{M^k})^{M'}(\omega) \leq (\tilde{Y}^i_{M^i})^{M'}(\omega)\right\}} dP(\omega) \\
&= \bigcap_{k \neq i} P^{M'}\left(\left\{\omega \in S_{M'} : (\tilde{Y}^k_{M^k})^{M'}(\omega) \leq (\tilde{Y}^i_{M^i})^{M'}(\omega)\right\}\right).
\end{aligned}
$$

Random variable $(\mathbf{1}^i)^{M'}$ is measurable with respect to the sigma-algebra $\bigcup_{i \in N}(\mathcal{I}^i_{M^i \cap M'})^{M'}$.

The first result characterizes the bidders' entry fees that the seller would optimally charge after committing to release information and awareness $(\mathcal{I}^1_{M^1}, \mathcal{I}^2_{M^2}, ..., \mathcal{I}^n_{M^n})$ as well the seller's expected revenue from optimal entry fees and the weak dominant strategy equilibrium of the second-price auction. Recall again that when the seller commits to release information $(\mathcal{I}^1_{M^1}, \mathcal{I}^2_{M^2}, ..., \mathcal{I}^n_{M^n})$, he makes bidder i aware of M^i, for $i = 1, ..., n$. Bidder i perceives that the seller committed to release information $(\mathcal{I}^1_{M^1 \cap M^i}, \mathcal{I}^2_{M^2 \cap M^i}, ..., \mathcal{I}^n_{M^n \cap M^i})$. That is, the seller privately communicates her commitment to information disclosure to each bidder. This communication is truthful w.r.t. the characteristics bidder i is made aware but it is silent on characteristics of which the seller keeps bidder i unaware.

Proposition 1 *If the seller commits to release the information $(\mathcal{I}^1_{M^1}, \mathcal{I}^2_{M^2}, ..., \mathcal{I}^n_{M^n})$ (and thereby raising bidder i's awareness to M^i), each bidder $i \in N$ is willing to pay an entry fee of*

$$
e^i = \mathbb{E}\left[\left(Y^i_{M^i} - (\tilde{Y}^{(2)})^{M^i}\right)(\mathbf{1}^i)^{M^i}\right] = \mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i}\right)(\mathbf{1}^i)^{M^i}\right].
$$

The expected revenue to the seller is

$$
\mathbb{E}[(\tilde{Y}^{(1)})^M] + \underbrace{\sum_{i \in N}\left(e^i - (e^i)^M\right)}_{\substack{\text{Unawareness rents} \\ \text{from bidders}}}
$$

where $(e^i)^M = \mathbb{E}[((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M)(\mathbf{1}^i)^M]$ is bidder i's entry fee from the point of view of an agent with full awareness M.

Proposition 1 is inspired by and generalizes Szech (2011). She considers the case in which all bidders are aware of all characteristics. In such a case, the seller's expected revenue is exactly $\mathbb{E}[(\tilde{Y}^{(1)})^M]$. In contrast, we allow the set of characteristics to be different among bidders. This leads to an additional term, $\sum_{i \in N}\left(e^i - (e^i)^M\right)$, in the seller's expected revenue, which is the sum of the differences between a bidder's actual

entry fee and the entry fee that the bidder should pay from the point of view of an agent with full awareness M. An agent with full awareness M realizes that bidder i with awareness M^i may have a biased perception of his probability winning and the expected price to be paid upon winning. This creates the difference between e^i and $(e^i)^M$. We may call $e^i - (e^i)^M$ the *unawareness rent* the seller accrues from agent i.

If a bidder is aware of all characteristics in M, then the unawareness rent is zero, i.e., $e^i - (e^i)^M = 0$. It follows that the rent $e^i - (e^i)^M$ accrues to the seller only from bidders who are unaware of some characteristic. Yet, asymmetry in awareness is also necessary for accrual of such rents to the seller. If every bidder is aware of the same set of characteristics, that is $M^i = M' \subseteq M$ for all $i \in N$, the term $\sum_{i \in N} \left(e^i - (e^i)^M \right)$ will drop out because $e^i = (e^i)^M$ for all $i \in N$ (even if $M' \subsetneq M$). Consequently, the seller's expected revenue is equal to the expected value of the first (highest) order statistics of estimated valuations across spaces. This is essentially Szech (2011)'s result except that in her work it is as if all bidders were aware of the full set of characteristics M to begin with.

Corollary 1 (Szech (2011)) *If the seller commits to release the information $(\mathcal{I}_{M'}^1, \mathcal{I}_{M'}^2, ..., \mathcal{I}_{M'}^n)$ and thereby raising every bidder's awareness to $M' \in \mathcal{M}$, then the seller's expected revenue is $\mathbb{E}[(\tilde{Y}^{(1)})^M]$.*

4 Optimally Raising Individual Awareness

So far, we characterized optimal entry fees given disclosure. Now we turn to optimal disclosure of individual awareness. Since optimal disclosure of information in auctions has already been studied elsewhere (e.g., Szech, 2011, Eso and Szentes, 2007, and Gerskov, 2009), we focus on optimally raising awareness. W.r.t. information, we assume that the seller has no control over information on the characteristic she may make bidders aware of. That is, when raising awareness of a characteristics, the seller may also provide information on this characteristic. However, while raising awareness of the characteristic is fully under the seller's control, we assume for this case that the seller cannot control the amount of information provided on the characteristics. This is for instance the case when the seller will have some information on some characteristics and raising awareness of a characteristic would also mandate her by law to eventually disclose her information on it. E.g., in real estate transactions in the US, sellers are required by law to disclose information on certain issues. On other issues, the seller is required to disclose all eventually available information *once the buyer asks for it*. Naturally, a buyer can only ask for information when he is aware of it or made aware of it. We will also assume that she will not provide further information on characteristics that bidders are already aware.

For any bidder $i \in N$ and characteristic $j \in M$, let μ_j^i denote the expected value of X_j^i. Our first observation is that given all the bidders are aware of the same set of

characteristics, the seller can improve her expected revenue by making one bidder aware of one more characteristic if the expected value of that characteristic is strictly positive.

Proposition 2 *Consider any $M' \in \mathcal{M}$ with $M' \subsetneq M$ and $M^i = M'$ for all $i \in N$, and let $\ell \in M \setminus M'$. If $\mu^1_\ell > 0$, then raising bidder 1's awareness of ℓ increases the seller's expected revenue.*

This observation follows from Proposition 1 and two lemmata. When all bidders are exactly aware of M', the seller's expected revenue is $\mathbb{E}[(\tilde{Y}^{(1)})^M]$. Let $\mathbb{E}[(\tilde{Z}^{(1)})^M]$ be the expected first order statistic when bidder 1 is made aware of characteristic ℓ. Moreover, let $\sum_{i \in N \setminus \{1\}} \left(e^i_z - (e^i_z)^M \right)$ be the sum of unawareness rents accrued to the seller from all other bidders upon making bidder 1 aware of ℓ. (Recall that in this case the unawareness rent from bidder 1 is zero.) According to Proposition 1 the seller's expected revenue upon raising bidder 1's awareness of ℓ is $\mathbb{E}[(\tilde{Z}^{(1)})^M] + \sum_{i \in N \setminus \{1\}} \left(e^i_z - (e^i_z)^M \right)$. We have $\mathbb{E}\left[(\tilde{Z}^{(1)})^M \right] > \mathbb{E}\left[(\tilde{Y}^{(1)})^M \right]$ by the following "standard" lemma:

Lemma 3 *Consider any $M' \in \mathcal{M}$ with $M' \subsetneq M$ and let $\ell \in M \setminus M'$. If $\mu^1_\ell > 0$, then*

$$\mathbb{E}[\max\{\tilde{Y}^1_{M'} + \tilde{X}^1_\ell, \tilde{Y}^2_{M'}, \ldots, \tilde{Y}^n_{M'}\}] \;>\; \mathbb{E}[\max\{\tilde{Y}^1_{M'}, \tilde{Y}^2_{M'}, \ldots, \tilde{Y}^n_{M'}\}].$$

Raising bidder 1's awareness of ℓ while keeping all other bidders unaware creates unawareness rents from bidders who remain unaware. The next lemma proved in the appendix states that these unawareness rents are positive when bidder 1's expected value of characteristic ℓ is positive.

Lemma 4 *Consider any $M' \in \mathcal{M}$ with $M' \subsetneq M$ and let $\ell \in M \setminus M'$. If $\mu^1_\ell > 0$, then*

$$\sum_{i \in N \setminus \{1\}} \left(e^i_z - (e^i_z)^M \right) > 0.$$

Together, these lemmata prove Proposition 2.

What about making further bidders aware of characteristic ℓ? Raising another bidder's awareness of characteristic ℓ involves now a trade-off. Suppose that bidders 1 to k with $k < n$ are all aware of characteristic ℓ already. If the expected value of the characteristic for bidder $k + 1$ is positive, i.e., $\mu^{k+1}_\ell > 0$, then raising awareness of ℓ increases the expected first order statistic, which is beneficial to the seller's expected revenue. It also increases the unawareness rents from bidders who remain unaware (if there are still bidders who are unaware) because they now overestimate their probability of winning and underestimate the price they have to pay upon winning even more. Again, this effect is beneficial to the seller. However, the seller also loses the unawareness rent from the bidder whom she made aware, which is the opportunity cost of raising the bidder's awareness of ℓ. The next observation states this trade-off of raising awareness more formally.

Proposition 3 *Let $M' \in \mathcal{M}$ with $M' \subsetneqq M$ and let $\ell \in M \setminus M'$. Suppose that bidders $\{1, ..., k\}$ are exactly aware of $M' \cup \{\ell\}$ while bidders $\{k+1, ..., n\}$ are exactly aware of M' only. Raising bidder $k + 1$'s awareness of characteristic ℓ increases the seller's expected revenue if $\mu_\ell^{k+1} > 0$ and*

$$\underbrace{\mathbb{E}\left[(\tilde{Z}^{(1)})^M\right] - \mathbb{E}\left[(\tilde{Y}^{(1)})^M\right]}_{\substack{\text{Increase in first order} \\ \text{statistics}}} + \underbrace{\sum_{i=k+2}^{n}\left((e^i)^M - (e_z^i)^M\right)}_{\substack{\text{Increase in unawareness} \\ \text{rents from remaining} \\ \text{unaware bidders}}} > \underbrace{e^{k+1} - (e^{k+1})^M}_{\substack{\text{Loss of unawareness} \\ \text{rent from bidder} \\ k+1}} \quad (1)$$

where $(\tilde{Y}^{(1)})^M$ and $(\tilde{Z}^{(1)})$ are the first order statistics before and after making bidder $k+1$ aware of ℓ, respectively, and e^i and e_z^i are the optimal entry fees of bidder i before and after making bidder $k+1$ aware of ℓ, respectively.

The first term of the l.h.s. of Inequality (1) measures the effect of raising one additional bidder's awareness of characteristic ℓ on the expected first order statistic. This effect is positive when $\mu_\ell > 0$ by the following "standard" lemma:

Lemma 5 *Let $M' \in \mathcal{M}$ with $M' \subsetneqq M$ and let $\ell \in M \setminus M'$. Suppose that bidders $\{1, ..., k\}$ are exactly aware of $M' \cup \{\ell\}$ while bidders $\{k+1, ..., n\}$ are exactly aware of M' only. If $\mu_\ell^{k+1} > 0$, then raising bidder $k + 1$'s awareness of characteristic ℓ yields*

$$\mathbb{E}\left[(\tilde{Z}^{(1)})^M\right] - \mathbb{E}\left[(\tilde{Y}^{(1)})^M\right] > 0.$$

The second term of the l.h.s. of Inequality (1), $\sum_{i=k+2}^{n}\left((e^i)^M - (e_z^i)^M\right)$, represents the increase of unawareness rents from remaining unaware bidders when bidder $k+1$ is made aware of the additional characteristic ℓ. Recall that unaware bidders "overpay" the participation fee because they overestimate their probability of winning and underestimate the amount they would have to pay in case of winning. When bidder $k+1$ is made aware of the additional characteristic ℓ with $\mu_\ell^{k+1} > 0$, then they overestimate their probability of winning even further (because their actual probability of winning decreases) and underestimate the amount they would have to pay in case of winning even further (because the actual expected second order statistic increases). The second term of the l.h.s. of Inequality (1) measures the increase of those overpayments, an increase of unawareness rents accrued to the seller from unaware bidders. We prove in the appendix:

Lemma 6 *Let $M' \in \mathcal{M}$ with $M' \subsetneqq M$ and let $\ell \in M \setminus M'$. Suppose that bidders $\{1, ..., k\}$ are exactly aware of $M' \cup \{\ell\}$ while bidders $\{k+1, ..., n\}$ are exactly aware of M' only. If $\mu_\ell^{k+1} > 0$, then raising bidder $k+1$'s awareness of characteristic ℓ yields*

$$\sum_{i=k+2}^{n}\left((e^i)^M - (e_z^i)^M\right) > 0.$$

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The r.h.s. of Inequality (1), $e^{k+1} - (e^{k+1})^M$, represents the loss of the unawareness rent from bidder $k + 1$ when he is made aware of characteristic ℓ. As proved in the appendix, the loss is positive when $\mu_\ell^i > 0$ for $i = 1, ..., k$:

Lemma 7 *Let $M' \in \mathcal{M}$ with $M' \subsetneq M$ and let $\ell \in M \setminus M'$. Suppose that bidders $\{1, ..., k\}$ are exactly aware of $M' \cup \{\ell\}$ while bidders $\{k + 1, ..., n\}$ are exactly aware of M' only. If $\mu_\ell^i > 0$ for $i = 1, ..., k$, then raising bidder $k + 1$'s awareness of characteristic ℓ yields*

$$e^{k+1} - (e^{k+1})^M > 0.$$

One may wonder whether $\mu_\ell^i > 0$ is necessarily for above observations to hold. In particular, for a seller to improve her revenue by making bidders aware of a characteristic without control of information, is it necessary that this characteristic has a non-negative expected value? The following examples provide negative answers.

Example 1. There are cases when the optimal awareness raising strategy of the seller raises all bidders' awareness of characteristic ℓ but $\mu_\ell < 0$. Suppose that for both bidders, the random variable for characteristic 1, X_1, is uniformly distributed on $[0, 5]$, and for characteristics 2, X_2, is uniformly distributed on $[-6, 5]$ with $\mu_2 = -\frac{1}{2}$. Then, $\mathbb{E}[\max\{X_1^1, X_1^2\}] = \frac{10}{3}$. The distribution of $Y = X_1 + X_2$ is

$$g(y) = \begin{cases} \frac{1}{55}(6 + y) & \text{if } -6 \leq y \leq -1, \\ \frac{1}{11} & \text{if } -1 < y \leq 5, \\ \frac{1}{55}(10 - y) & \text{if } 5 < y \leq 10. \end{cases}$$

Suppose the seller would disclose to bidder 1 the first characteristic with full information. Then we have

$$\begin{aligned}
& \mathbb{E}\left[\max\{X_1^1 + X_2^1, X_1^2\}\right] \\
= {} & \int_0^5 \int_{x_1}^{10} y g(y) \, \mathrm{d}y f(x_1) \, \mathrm{d}x_1 + \int_{-6}^0 \int_0^5 x_1 f(x_1) \, \mathrm{d}x_2 g(y) \, \mathrm{d}y + \int_0^5 \int_y^5 x_1 f(x_1) \, \mathrm{d}x_1 g(y) \, \mathrm{d}y \\
= {} & \frac{505}{132} \approx 3.826,
\end{aligned}$$

which is greater than $\mathbb{E}[\max\{X_1^1, X_1^2\}]$. This example just shows that there are cases when the expected value of the second characteristic is negative, we can still have $\mathbb{E}[\max\{X_1^1 + X_2^1, X_1^2\}] > \mathbb{E}[\max\{X_1^1, X_1^2\}]$. $\qquad\square$

The next example shows that the previous counterexample is not pathological. We obtain similar examples in the "normal" case.

Example 2. Suppose that for both bidders, the random variables for characteristics 1 and 2, X_1 and X_2, are independent normal random variables distributed as $N(\mu_1, \sigma_1^2)$

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and $N(\mu_2, \sigma_2^2)$. Then $X_1 + X_2$ is distributed as $N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$. Applying the results from Clark (1961), we have the expected values for the two highest order statistics as follows:

$$\mathbb{E}[\max\{X_1^1, X_1^2\}] = \mu_1 + \frac{\sigma_1}{\sqrt{\pi}}$$

$$\mathbb{E}[\max\{X_1^1 + X_2^1, X_1^2\}] = \mu_1 + \mu_2 \Phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) + \sqrt{2\sigma_1^2 + \sigma_2^2}\,\phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right),$$

where Φ and ϕ are the CDF and PDF of a standard normal distribution. The difference between $\mathbb{E}[\max\{X_1^1 + X_2^1, X_1^2\}]$ and $\mathbb{E}[\max\{X_1^1, X_1^2\}]$, which is equal to $\mu_2 \Phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) + \sqrt{2\sigma_1^2 + \sigma_2^2}\,\phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) - \frac{\sigma_1}{\sqrt{\pi}}$, does not depend on μ_1, but it increases in μ_2 and σ_2, and decreases in σ_1.[3] When $\mu_2 = 0$, the difference becomes

$$\frac{\sqrt{2\sigma_1^2 + \sigma_2^2}}{\sqrt{2\pi}} - \frac{\sigma_1}{\sqrt{\pi}} \geq 0$$

with equality when $\sigma_2 = 0$, which would make X_2 a constant. (This case is ruled out almost surely by assumption). By monotonicity, if $\mu_2 > 0$,

$$
\begin{aligned}
&\mathbb{E}[\max\{X_1^1 + X_2^1, X_1^2\}] - \mathbb{E}[\max\{X_1^1, X_1^2\}] \\
=\quad & \mu_2 \Phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) + \sqrt{2\sigma_1^2 + \sigma_2^2}\,\phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) - \frac{\sigma_1}{\sqrt{\pi}} \\
>\quad & \frac{\sqrt{2\sigma_1^2 + \sigma_2^2}}{\sqrt{2\pi}} - \frac{\sigma_1}{\sqrt{\pi}}.
\end{aligned}
$$

In a two-bidder setting, $\mu_2 > 0$ is a sufficient condition for the inequality. The necessary condition also depends on σ_1 and σ_2. In the extreme case when $\sigma_1 = 0$ and $\sigma_2 = 0$ (and thus information about those random variables would not play any role), $\mu_2 > 0$ is the sufficient and necessary condition. (Again, we ruled out earlier by assumption that random variables are almost surely constant.) $\qquad\square$

5 Optimally Raising Common Awareness

So far, we considered a seller who can make bidders aware individually via targeted disclosure of awareness to a particular bidder. Now we consider the case where the seller

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[3]Let D be the difference. We have $\frac{\partial D}{\partial \mu_1} = \Phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) > 0$, $\frac{\partial D}{\partial \sigma_2} = \frac{\sigma_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) > 0$, and $\frac{\partial D}{\partial \sigma_1} = \frac{2\sigma_1}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) - \frac{1}{\sqrt{\pi}} < 0$ since $\frac{2\sigma_1}{\sqrt{2\sigma_1^2 + \sigma_2^2}} < \sqrt{2}$ and $\phi\left(\frac{\mu_2}{\sqrt{2\sigma_1^2 + \sigma_2^2}}\right) < \frac{1}{\sqrt{2\pi}}$.

is constrained to disclose awareness to *all* bidders if any. In such a case, unawareness rents are always zero by Corollary 1. That is, awareness is never asymmetric. We focus on two sub-cases: First, we consider the case of raising common awareness of a characteristic without providing any information on it. This is followed by a second case in which the seller raises common awareness of a characteristic and provides full information about the characteristic.

5.1 No Information

Raising awareness of a characteristics without providing any information on the characteristic is relevant for instance, when bidders bid for access to a user and the seller is forbidden by privacy laws to provide any information about the user. In such a case, the seller could at least raise the bidders' awareness of all characteristics that she collects information on without disclosing any of this information. Subsequently, bidders may now consider the expected value of some characteristics that they did not take into account previously. Another instance would be a procurement setting in which the procurement officer raises awareness of potential events that might affect the bidders' costs without having any information on these events. Yet, another example would be the disclosure of risk factors that might effect the future profitability of a take over candidate in financial markets. Typically, such risk factors are just listed in official fillings without information as to their likelihood.

We assume for this subsection that the valuations for each characteristic are not only independently distributed across bidders but also identically distributed across bidders. For any characteristic $\ell \in M$, let μ_ℓ denote the (common) expected value of X_ℓ.

Proposition 4 *Suppose the seller can only commit to disclosure of information $(\mathcal{I}_{M'}^1, \mathcal{I}_{M'}^2, ..., \mathcal{I}_{M'}^n)$ that is completely uninformative about characteristics M' but allows for raising common awareness of any subsets of characteristics $M' \in \mathcal{M}$. For any characteristic $\ell \in M$, the optimal common awareness raising strategy of the seller raises all bidders' awareness of characteristic ℓ if and only if $\mu_\ell > 0$.*

The proof is in Appendix B.

5.2 Full Information

Raising common awareness under full information is relevant in cases where the seller will eventually have full information and is required by law to disclose all information. Bidders can only ask for that information (after paying their entry fees) when they are aware of it or made aware of it. The next proposition characterizes raising awareness under full information.

Proposition 5 *Suppose the seller can commit to any disclosure of information $(\mathcal{I}_{M'}^1, \mathcal{I}_{M'}^2, ..., \mathcal{I}_{M'}^n)$ with common awareness of characteristics in M' for any $M' \in \mathcal{M}$ but full information about characteristics disclosed. For any $\ell \in M$, the optimal awareness raising strategy of the seller keeps all bidders unaware of ℓ if and only if*

$$\mathbb{E}[\max\{X_\ell^1, X_\ell^2, \ldots, X_\ell^n\}] < 0.$$

In order to related this result to the prior observation, note that if $\{X_\ell^i\}_{i \in N}$ are identically distributed such that $\mu_\ell^i = \mu_\ell$, then $\mu_\ell < 0$ is still a necessary condition for keeping bidders unaware of characteristic ℓ under full information since $\mathbb{E}[\max\{X_\ell^1, X_\ell^2, \ldots, X_\ell^n\}] < 0$ implies $\mu_\ell < 0$.

6 Discussion

6.1 Winner's Curse Due to Unawareness

If the seller raises optimally awareness of characteristics, then any characteristic she keeps bidders unaware has a negative expected value. This will lead to an effect akin to the winner's curse even. This winner's curse is not due to common values but due to unawareness of characteristics with negative expected value and occurs even in our setting with independent valuations.

To illustrate, suppose that all bidders are aware of characteristics in the set $M' \in \mathcal{M}$ and there is one characteristic $\ell \in M \setminus M'$ represented by a random variable X_ℓ that is independently and identically distributed among all bidders. Assume that $\mu_\ell < 0$. If the seller keeps bidders unaware of ℓ, her expected revenue will be $\mathbb{E}[Y^{(1)}]$ such that for all $i \in N$, $Y^i = \sum_{j \in M'} X_j^i$ is based on the same set of characteristics M' that does not include ℓ. If bidder i were aware of ℓ, her valuation of the object would be $Y_{M'}^i + X_\ell$. In such a case, the winner's actual expected payoff from the auction would be $\mathbb{E}[Y^{(1)}] - \mathbb{E}[Y^{(2)}] - e^i + \mathbb{E}[X_\ell]$, which differs from her perceived expected payoff by $\mathbb{E}[X_\ell] = \mu_\ell$. Since $\mu_\ell < 0$, the winner's actual expected payoff is lower than the winner's perceived expected payoff. This is a form of winner's curse due to unawareness of characteristics that negatively effect bidders.

Bidders may realize that the seller has no incentive to raise awareness of characteristics that negatively effect bidders. That is, they may become aware that there may or may not be characteristics of which they are unaware. Further, they may realize that if there is such a characteristic, then it has a negative expected value. Each bidder may form different beliefs about what he might be unaware of and adjusts his bids accordingly. At the end the most optimistic among the bidders might win the auction and may still suffer a winner's curse.

We emphasize that the winner's curse in our setting is different from the standard setting. The winner's course in standard common value auctions occurs because the

winner is the interim most optimistic bidder even with bidders that are unbiased ex ante. Our case features independent values. Bidders are also unbiased w.r.t. characteristics they are aware of but because the auctioneer keeps bidders unaware of characteristics with negative expected value, they essentially misperceive their expected value of the object.

6.2 How is Awareness Different from Information?

Conceptually, awareness is very different from information. Clearly, there is a difference between not being able to think of a random variable X_ℓ (aka unawareness of ℓ) and realizing that there is a random variable X_ℓ but having no information about it and thus only taking into account μ_ℓ (aka lack of information about X_ℓ).

In the context of second-price auctions with costless information, disclosure of information is very different from disclosure of awareness. We have seen that disclosure of awareness involves a delicate trade-off between the effect on the expected first order statistic and the effect on unawareness rents. Optimal disclosure of costless information on the other hand leads to full disclosure. In our context, given common awareness of a set of characteristics, full information disclosure is optimal to the seller.

Proposition 6 *Given a set of characteristics $M' \in \mathcal{M}$, suppose the seller can commit to any disclosure of information $(\mathcal{I}_{M'}^1, \mathcal{I}_{M'}^2, ..., \mathcal{I}_{M'}^n)$ (i.e., any information structure for which all bidders' awareness are raised to a given common set of characteristics $M' \in \mathcal{M}$). Then committing to full information on all characteristics in M' maximizes the seller's expected revenue among all $(\mathcal{I}_{M'}^1, \mathcal{I}_{M'}^2, ..., \mathcal{I}_{M'}^n)$.*

While the result generalizes an observation Szech (2011) for situations of common full awareness, the arguments in the appendix are the same as in Szech (2011). Similar results and arguments have also been presented by Eso and Szentes (2007) and Gerskov (2009).

6.3 Related Literature on Unawareness

Beside to the literature on information disclosure in auctions, we contribute to the study of unawareness in contracting. We make use of unawareness type spaces, albeit in a very restricted form, by Heifetz, Meier, and Schipper (2013a), Bayesian games with unawareness by Meier and Schipper (2014), and dynamic games with unawareness by Heifetz, Meier, and Schipper (2013b). Francetich and Schipper (2024) study a screening problem in which the agent is more aware than the principal and has to decide whether or not to raise the principal's awareness before the principal offers a menu of screening contracts. They show that when the principal is unaware of high marginal cost types only, then the agent happily raises her awareness while this is not the case when the principal is

unaware of low marginal cost types only. Herweg and Schmidt (2020) study a procurement problem in which the seller may be aware of some ex post verifiable design flaws . They propose an efficient two-stage mechanism with a neutral arbitrator that does not require an ex ante description of design flaws. Grant, Kline, and Quiggin (2012) discuss disagreements arising from asymmetric awareness in contracting. Pram and Schipper (2024) study general efficient mechanism design under unawareness for quasi-linear environments. They devise dynamic elaboration direct mechanisms for efficient implementation with no-deficit. von Thadden and Zhao (2012) study a principal-agent moral hazard problem in which the principal is aware of actions of which the agent is unaware. When contemplating whether or not to make the agent aware, the principal faces a trade-off between getting a better action and saving on information rents due to additional incentive compatibility constraints. Auster (2013) studies a principal-agent moral hazard problem in which the principal is aware of contingencies of which the agent is unaware but whose realization is informative about the agent's actions. In the optimal contract, the principal faces a trade-off between exploiting the agent's unawareness and using said contingencies in order to provide incentives. Filiz-Ozbay (2012) studies a risk neutral insurer who is aware of some contingencies that the insuree is unaware. The insurer has an incentive to mention only some contingencies in a contract while remaining silent on others. Auster and Pavoni (2024) and Lei and Zhao (2021) feature an agent with higher awareness level than the principal but in the context of optimal delegation. In Auster and Pavoni (2024), the agent is aware of both the set of his actions and their performance, and only reveals extreme actions. In Lei and Zhao (2021), the agent only partially reveals contingencies of which the principal is unaware. Principals who are unaware of more contingencies delegate a large set of projects. Finally, in the field of value-based business strategy, Bryan, Ryall, and Schipper (2021) devise cooperative games with incomplete information and unawareness for studying surplus creation and surplus appropriation within contracting relationships.

A   Additional Mathematical Results for the Model

This appendix collects some mathematical results that are useful for applying our model using specific distributions.

Denote by $(G_{M^i}^i)^{M'}$ the distribution of $Y_{M^i}^i$ from the point of view of a player who is aware of M'. That is,

$$(G_{M^i}^i)^{M'}(y^i) = P^{M'}\left(\left\{\omega \in S_{M'} : Y_{M^i}^i(\omega) \leq y^i\right\}\right).$$

We like to obtain an expression for $(G_{M^i}^i)^{M'}$ in terms of the distributions of X_j^i's. To this extent, define an isotone bijection $b^{M^i \cap M'} : M^i \cap M' \longrightarrow \{1, ..., |M^i \cap M'|\}$ by $b^{M^i \cap M'}(1) = 1$, and $b^{M^i \cap M'}(j) > b^{M^i \cap M'}(k)$ if and only if $j > k$. This bijection just renumbers characteristics in $M^i \cap M'$ consecutively. Using this renumbering and the

assumption of independence of random variables across characteristics, the following lemma follows directly from the convolution theorem (see for instance, Chung, 2001, Theorem 6.1.1., Corollary p. 153, and Theorem 6.1.4.):

Lemma 8

$$(G_{M^i}^i)^{M'}(y^i) \;=\; \int \cdots \int F_1^i \left(y^i - x_2^i - \ldots - x_{|M^i \cap M'|}^i \right) dF_2^i \left(x_2^i \right) \cdots dF_{|M^i \cap M'|}^i \left(x_{|M^i \cap M'|}^i \right)$$

Denote the distribution of $\tilde{Y}_{M^i}^i$ from the point of view of a player with awareness M' given $((\mathcal{I}_{M^i \cap M'}^i)^{M'})_{i \in N}$ by $(\tilde{G}_{M^i}^i)^{M'}$ such that

$$(\tilde{G}_{M^i}^i)^{M'}(y^i) = P^{M'}(\{\omega \in S_{M'} : (\tilde{Y}_{M^i}^i)^{M'}(\omega) \le y^i\}).$$

Denote the probability distributions of $(\tilde{Y}^{(1)})^{M'}$ and $(\tilde{Y}^{(2)})^{M'}$ on $S_{M'}$ given $((\mathcal{I}_{M^i \cap M'}^i)^{M'})_{i \in N}$ by $(\tilde{G}^{(1)})^{M'}$ and $(\tilde{G}^{(2)})^{M'}$, respectively. The characterizations of these distributions follow from Güngör, Bulut, and Çahk (2009, Result 2.4):

Lemma 9

$$(\tilde{G}^{(1)})^{M'}(y) \;=\; \prod_{i \in N}(\tilde{G}_{M^i}^i)^{M'}(y)$$

$$(\tilde{G}^{(2)})^{M'}(y) \;=\; \prod_{i \in N}(\tilde{G}_{M^i}^i)^{M'}(y) + \sum_{i \in N} \left[(1 - (\tilde{G}_{M^i}^i)^{M'}(y)) \prod_{k \in N \setminus \{i\}} (\tilde{G}_{M^k}^k)^{M'}(y) \right]$$

PROOF.   Recall that the permanent of a $n \times n$ square matrix $\mathbf{A} = (a_{ij})$ is defined by

$$per\mathbf{A} = \sum_{\sigma \in P_n} \prod_{i=1}^{n} a_{i\sigma(i)},$$

where P_n is the set of permutations of $\{1, 2, \ldots, n\}$. Then the permanent is equivalent to the determinant except that all signs in the expansion are positive. From Result 2.4 of Güngör, Bulut, and Çahk (2009) for the distribution function of order statistics with ascending values, we have the distribution function for the $(n + 1 - r)$-th order statistic for descending values

$$(\tilde{G}^{(n+1-r)})^{M'}(y) \;=\; \Pr[(\tilde{Y}^{(n+1-r)})^{M'} \le y]$$

$$=\; \sum_{m=r}^{n} \frac{1}{m!(n-m)!} per \left[\underbrace{(\tilde{\mathbf{G}})^{M'}(y)}_{m} \; \underbrace{\mathbf{1} - (\tilde{\mathbf{G}})^{M'}(y)}_{n-m} \right]$$

where $(\tilde{\mathbf{G}})^{M'}(y) := \left((\tilde{G}_{M^1}^1)^{M'}(y), (\tilde{G}_{M^2}^2)^{M'}(y), \ldots, (\tilde{G}_{M^n}^n)^{M'}(y) \right)'$ and $\mathbf{1} - (\tilde{\mathbf{G}})^{M'}(y) :=$ $\left(1 - (\tilde{G}_{M^1}^1)^{M'}(y), 1 - (\tilde{G}_{M^2}^2)^{M'}(y), \ldots, 1 - (\tilde{G}_{M^n}^n)^{M'}(y) \right)'$ are column vectors, and there

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are m columns of $(\tilde{\mathbf{G}})^{M'}(y)$ and $n - m$ columns of $\mathbf{1} - (\tilde{\mathbf{G}})^{M'}(y)$ in the *permanent* operator. Therefore, when $r = n$ we have the distribution function for the highest order statistic

$$
\begin{aligned}
(\tilde{G}^{(1)})^{M'}(y) &= \Pr[(\tilde{Y}^{(1)})^{M'} \leq y] \\
&= \frac{1}{n!0!} per \left[\underbrace{(\tilde{\mathbf{G}})^{M'}(y)}_{n} \right] \\
&= \prod_{i \in N} (\tilde{G}_{M^i}^i)^{M'}(y),
\end{aligned}
$$

and when $r = n-1$ we have the distribution function for the second-highest order statistic

$$
\begin{aligned}
(\tilde{G}^{(2)})^{M'}(y) &= \Pr[(\tilde{Y}^{(2)})^{M'} \leq y] \\
&= \sum_{m=n-1}^{n} \frac{1}{m!(n-m)!} per \left[\underbrace{(\tilde{\mathbf{G}})^{M'}(y)}_{m} \; \underbrace{\mathbf{1} - (\tilde{\mathbf{G}})^{M'}(y)}_{n\text{-}m} \right] \\
&= \frac{1}{(n-1)!1!} per \left[\underbrace{(\tilde{\mathbf{G}})^{M'}(y)}_{n\text{-}1} \; \underbrace{\mathbf{1} - (\tilde{\mathbf{G}})^{M'}(y)}_{1} \right] + \frac{1}{n!0!} per \left[\underbrace{(\tilde{\mathbf{G}})^{M'}(y)}_{n} \right] \\
&= \sum_{i \in N} \left[(1 - (\tilde{G}_{M^i}^i)^{M'}(y)) \prod_{k \in N \setminus \{i\}} (\tilde{G}_{M^k}^k)^{M'}(y) \right] + \prod_{i \in N} (\tilde{G}_{M^i}^i)^{M'}(y).
\end{aligned}
$$

$\square$

B   Proofs

Proof of Lemma 1

Since $X_{M'}$ is a measurable random variable on $S_{M'}$, we have for every $x_{M'} \in \mathbb{R}^{n \cdot |M'|}$ that $\{\omega \in S_{M'} : X_{M'}(\omega) \leq x_{M'}\} \in \mathcal{F}_{M'}$. From the extension of $X_{M'}$ to $S_{M''}$ follows that $\{\omega \in S_{M'} : X_{M'}((r_{M'}^{M''})^{-1}(\omega)) \leq x_{M'}\} \in \mathcal{F}_{M'}$. Since projections are measurable, we have $(r_{M'}^{M''})^{-1} \left(\{\omega \in S_{M'} : X_{M'}((r_{M'}^{M''})^{-1}(\omega)) \leq x_{M'}\} \right) \in \mathcal{F}_{M''}$, which is equivalent to $\{\omega \in S_{M''} : X_{M'}(\omega) \leq x_{M'}\} \in \mathcal{F}_{M''}$. $\square$

Proof of Lemma 2

For any $M', M'', M''' \in \mathcal{M}$ with $M' \subseteq M'' \cap M'''$ and any $x_{M'} \in \mathbb{R}^{n \cdot |M'|}$, we have

$$
\begin{aligned}
P^{M'' \cap M'''}&(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}) \\
&= P^{M''}_{|M'' \cap M'''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}) && (2) \\
&= P^{M''}\left((r^{M''}_{M'' \cap M'''})^{-1}(\{\omega \in S_{M'} : X_{M'}(\omega) \leq x_{M'}\})\right) && (3) \\
&= P^{M''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\} \cap S_{M''}) && (4) \\
&= P^{M''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}), && (5)
\end{aligned}
$$

where equation (2) follows from the fact that P is a projective system of probability measures, equation (3) follows from the definition of marginal, and equation (4) follows from the extension of the random variable $X_{M'}$ to $S_{M''}$. By analogous arguments, we have $P^{M'' \cap M'''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}) = P^{M'''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\})$. Hence $P^{M''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\}) = P^{M'''}(\{\omega \in \Omega : X_{M'}(\omega) \leq x_{M'}\})$. $\qquad \square$

Proof of Proposition 1

The entry fee for bidder i is her expected payoff from the weak dominant strategy equilibrium of the second-price auction with bidders informed by $((\mathcal{I}^k_{M^k \cap M^i})^{M^i})_{k \in N}$. We want to show that it is exactly equal to $e^i = \mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i}\right)(\mathbf{1}^i)^{M^i}\right]$. We have

$$
\begin{aligned}
\mathbb{E}\left[(\tilde{Y}^{(1)})^{M^i}(\mathbf{1}^i)^{M^i}\right] &= \mathbb{E}\left[(\tilde{Y}^i_{M^i})^{M^i}(\mathbf{1}^i)^{M^i}\right] \\
&= \mathbb{E}\left[\mathbb{E}[Y^i_{M^i} \mid (\mathcal{I}^i_{M^i})^{M^i}](\mathbf{1}^i)^{M^i}\right] \\
&= \mathbb{E}\left[\mathbb{E}[Y^i_{M^i} \mid \cup_{k \in N}(\mathcal{I}^k_{M^k \cap M^i})^{M^i}](\mathbf{1}^i)^{M^i}\right] \\
&= \mathbb{E}\left[\mathbb{E}[Y^i_{M^i}(\mathbf{1}^i)^{M^i} \mid \cup_{k \in N}(\mathcal{I}^k_{M^k \cap M^i})^{M^i}]\right] \\
&= \mathbb{E}\left[Y^i_{M^i}(\mathbf{1}^i)^{M^i}\right],
\end{aligned}
$$

where the first equality follows because bidder i's estimated valuation is the highest if he wins the auction, the second equality is by the definition of $(\tilde{Y}^i_{M^i})^{M^i}$, the third equality is due to independence, the fourth equality follows from $(\mathbf{1}^i)^{M^i}$ being already conditioned on $((\mathcal{I}^k_{M^k \cap M^i})^{M^i})_{k \in N}$, and the last equality follows from the law of iterated expectations. Therefore, together with the linearity of the expectations operator we have

$$
\mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i}\right)(\mathbf{1}^i)^{M^i}\right] = \mathbb{E}\left[\left(Y^i_{M^i} - (\tilde{Y}^{(2)})^{M^i}\right)(\mathbf{1}^i)^{M^i}\right] = e^i.
$$

The seller's expected revenue is the sum of entry fees from all bidders plus the expected selling price in the second-price auction. The expected selling price in the second-price

auction is the expectation of second highest order statistics across all bidders' estimated valuations from the seller's point of view. Therefore, the seller's expected revenue is $\sum_{i \in N} e^i + \mathbb{E}[(\tilde{Y}^{(2)})^M]$. We can further write out the expression as:

$$\sum_{i \in N} e^i + \mathbb{E}[(\tilde{Y}^{(2)})^M]$$

$$= \mathbb{E}[(\tilde{Y}^{(1)})^M] + \sum_{i \in N} e^i - \left[\mathbb{E}[(\tilde{Y}^{(1)})^M] - \mathbb{E}[(\tilde{Y}^{(2)})^M] \right]$$

$$= \mathbb{E}[(\tilde{Y}^{(1)})^M] + \sum_{i \in N} \mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i} \right) (\mathbf{1}^i)^{M^i} \right] - \mathbb{E}\left[(\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M \right]$$

$$= \mathbb{E}[(\tilde{Y}^{(1)})^M] + \sum_{i \in N} \mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i} \right) (\mathbf{1}^i)^{M^i} \right] - \mathbb{E}\left[\sum_{i \in N} ((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M)(\mathbf{1}^i)^M \right]$$

$$= \mathbb{E}[(\tilde{Y}^{(1)})^M] + \sum_{i \in N} \mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i} \right) (\mathbf{1}^i)^{M^i} \right] - \sum_{i \in N} \mathbb{E}\left[\left((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M \right) (\mathbf{1}^i)^M \right]$$

$$= \mathbb{E}[(\tilde{Y}^{(1)})^M] + \sum_{i \in N} \left(e^i - (e^i)^M \right)$$

$$\square$$

Proof of Corollary 1

If every bidder is aware of the same set of characteristics, $M^i = M' \subseteq M$ for all $i \in N$, then $M^i \cap M' = M \cap M' = M'$ and $\left(\mathcal{I}^i_{M^i \cap M'} \right)_{i \in N} = \left(\mathcal{I}^i_{M^i \cap M} \right)_{i \in N}$. Hence, $\mathbb{E}[(\tilde{Y}^{(1)})^{M^i}] = \mathbb{E}[(\tilde{Y}^{(1)})^M]$ and $\mathbb{E}[(\tilde{Y}^{(2)})^{M^i}] = \mathbb{E}[(\tilde{Y}^{(2)})^M]$ for all $i \in N$. Therefore, the sum of the entry fees from the bidder can be written as

$$\sum_{i \in N} e^i = \sum_{i \in N} \mathbb{E}\left[\left((\tilde{Y}^{(1)})^{M^i} - (\tilde{Y}^{(2)})^{M^i} \right) (\mathbf{1}^i)^{M^i} \right]$$

$$= \sum_{i \in N} \mathbb{E}\left[\left((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M \right) (\mathbf{1}^i)^M \right]$$

$$= \mathbb{E}\left[\sum_{i \in N} \left((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M \right) (\mathbf{1}^i)^M \right]$$

$$= \mathbb{E}\left[(\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M \right],$$

where the first equality follows from Proposition 1, the second equality is based on the condition that every bidder is aware of the same set of characteristics, and the third equality follows from the linearity of the expectations operator. Then the seller's expected revenue is $\mathbb{E}[(\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M] + \mathbb{E}[(\tilde{Y}^{(2)})^M] = \mathbb{E}[(\tilde{Y}^{(1)})^M]$. $\square$

Proof of Lemma 4

For the term $\sum_{i \in N \setminus \{1\}} \left(e_z^i - (e_z^i)^M \right)$, every bidder $i \in N \setminus \{1\}$ believes that all the bidders are aware of the same set of characteristics, so the entry fee e_z^i is computed based on this profile of information sets. However, the optimal entry fee from the point of view of an agent with full awareness M, $(e_z^i)^M$, considers the fact that bidder 1 is aware of characteristic ℓ in addition to the characteristics in M'. We claim that for all $i \neq 1$,

$$e_z^i > (e_z^i)^M$$
$$\mathbb{E}\left[((\tilde{Z}^{(1)})^{M^i} - (\tilde{Z}^{(2)})^{M^i})(\mathbf{1}_z^i)^{M^i} \right] > \mathbb{E}\left[((\tilde{Z}^{(1)})^M - (\tilde{Z}^{(2)})^M)(\mathbf{1}_z^i)^M \right]$$

We distinguish two cases: First, conditional on $k \neq i$ winning the object, we have

$$\mathbb{E}\left[((\tilde{Z}^{(1)})^{M^i} - (\tilde{Z}^{(2)})^{M^i})(\mathbf{1}_z^i)^{M^i} \mid k \text{ wins} \right] = \mathbb{E}\left[((\tilde{Z}^{(1)})^M - (\tilde{Z}^{(2)})^M)(\mathbf{1}_z^i)^M \mid k \text{ wins} \right]$$

simply because both $(\mathbf{1}_z^i)^{M^i}$ and $(\mathbf{1}_z^i)^M$ must take the value of 0 conditional on $k \neq i$ winning.

Second, if $\mu_\ell^1 > 0$, then conditional on i winning the object (recall that $i \neq 1$), we have

$$\mathbb{E}\left[((\tilde{Z}^{(1)})^{M^i} - (\tilde{Z}^{(2)})^{M^i})(\mathbf{1}_z^i)^{M^i} \mid i \text{ wins} \right] > \mathbb{E}\left[((\tilde{Z}^{(1)})^M - (\tilde{Z}^{(2)})^M)(\mathbf{1}_z^i)^M \mid i \text{ wins} \right].$$

To see this, we distinguish two sub-cases. Conditional on 1 not being the bidder with the second highest valuation, the expectations above are equal because the additional component that bidder 1 is aware of is not relevant for either the first order or second order statistics. Conditional on 1 being the bidder with the second highest valuation, we must have that the expectation of the second order statistic from the point of view of an agent with awareness M^i is less than the one from the point of view of an agent with awareness M because latter is aware of characteristic ℓ and $\mu_\ell^1 > 0$. This argument proves the claim. Therefore, if $\mu_\ell^1 > 0$, then we have $\sum_{i \in N \setminus \{1\}} \left(e_z^i - (e_z^i)^M \right) > 0$. $\qquad \square$

Proof of Proposition 3

Suppose the set of bidders in $\{1, 2, \ldots, k\}$ with $k < n$ are aware of the set of characteristics M' and characteristic $\ell \in M \setminus M'$ while the rest of the bidders are aware of M' only. By Proposition 1, the seller's expected revenue is $\mathbb{E}[(\tilde{Y}^{(1)})^M] + \sum_{i=k+1}^n \left(e^i - (e^i)^M \right)$ where w.l.o.g. we can single out bidder $k+1$'s difference in entry fees and rewrite the expected revenue as

$$\mathbb{E}\left[(\tilde{Y}^{(1)})^M \right] + e^{k+1} - (e^{k+1})^M + \sum_{i=k+2}^n \left(e^i - (e^i)^M \right).$$

Suppose now that bidder $k+1$ becomes aware of characteristic ℓ. The seller's expected revenue in this case are by Proposition 1,

$$\mathbb{E}[(\tilde{Z}^{(1)})^M] + \sum_{i=k+2}^{n} \left(e_z^i - (e_z^i)^M\right).$$

Note that for all $i = k+2, ..., n$, $e^i = e_z^i$. Thus,

$$\mathbb{E}[(\tilde{Z}^{(1)})^M] + \sum_{i=k+2}^{n} \left(e_z^i - (e_z^i)^M\right)$$

$$> \quad \mathbb{E}\left[(\tilde{Y}^{(1)})^M\right] + e^{k+1} - (e^{k+1})^M + \sum_{i=k+2}^{n} \left(e^i - (e^i)^M\right)$$

is equivalent to

$$\mathbb{E}[(\tilde{Z}^{(1)})^M] - \mathbb{E}\left[(\tilde{Y}^{(1)})^M\right] \quad > \quad e^{k+1} - (e^{k+1})^M + \sum_{i=k+2}^{n} \left((e_z^i)^M - (e^i)^M\right)$$

$\square$

Proof of Lemma 6

Let $M' \in \mathcal{M}$ with $M' \subsetneq M$ and let $\ell \in M \setminus M'$. Suppose that bidders $\{1, ..., k\}$ are exactly aware of $M' \cup \{\ell\}$ while bidders $\{k+1, ..., n\}$ are exactly aware of M' only, with $\ell \in M \setminus M'$. Assume that $\mu_\ell^1 > 0$. When raising bidder $k+1$'s awareness of ℓ, then for any $i = k+2, ..., n$,

$$(e^i)^M \quad > \quad (e_z^i)^M$$
$$\mathbb{E}\left[((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M)(\mathbf{1}^i)^M\right] \quad > \quad \mathbb{E}\left[((\tilde{Z}^{(1)})^M - (\tilde{Y}^{(Z)})^M)(\mathbf{1}_z^i)^M\right]$$

When i wins in the weak dominant strategy equilibrium of the second-price auction, then $(\tilde{Y}^{(1)})^M = (\tilde{Y}_{M'}^i)^M$. In this case, we also have $(\tilde{Y}^{(2)})^M = \max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+1}, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^{i-1}, \tilde{Y}_{M'}^{i+1}, ...\tilde{Y}_{M'}^n\}$. Similarly, if i wins in the weak dominant strategy equilibrium of the second-price auction after bidder $k+1$ was made aware of characteristic ℓ, then $(\tilde{Z}^{(1)})^M = (\tilde{Y}_{M'}^i)^M$ and $(\tilde{Z}^{(2)})^M = \max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+1} + \tilde{X}_\ell^{k+1}, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^{i-1}, \tilde{Y}_{M'}^{i+1}, ...\tilde{Y}_{M'}^n\}$. Thus, above inequality becomes

$$\mathbb{E}\left[((\tilde{Y}_{M'}^i)^M - \max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+1}, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^{i-1}, \tilde{Y}_{M'}^{i+1}, ...\tilde{Y}_{M'}^n\})(\mathbf{1}^i)^M\right]$$

$$> \quad \mathbb{E}\left[((\tilde{Y}_{M'}^i)^M - \max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+1} + \tilde{X}_\ell^{k+1}, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^{i-1}, \tilde{Y}_{M'}^{i+1}, ...\tilde{Y}_{M'}^n\})(\mathbf{1}_z^i)^M\right]$$

The inequality follows now from three facts: First, on the r.h.s. we have $\tilde{Y}_{M'}^{k+1} + \tilde{X}_\ell^{k+1}$ under the max function while we have only $\tilde{Y}_{M'}^{k+1}$ at the l.h.s. Second, $\mu_\ell^{k+1} > 0$. Third, the probability of bidder i winning is lower when bidder $k+1$ is aware of characteristic ℓ (r.h.s.) compared to when bidder $k+1$ is only aware of M' (l.h.s.) if $\mu_\ell^{k+1} > 0$. $\square$

Proof of Lemma 7

Let $M' \in \mathcal{M}$ with $M' \subsetneq M$ and let $\ell \in M \setminus M'$. Suppose that bidders $\{1, ..., k\}$ are exactly aware of $M' \cup \{\ell\}$ while bidders $\{k+1, ..., n\}$ are exactly aware of M' only, with $\ell \in M \setminus M'$. Assume that $\mu_\ell^i > 0$ for $i = 1, ..., k$.

$$e^{k+1} > (e^{k+1})^M$$

$$\mathbb{E}\left[((\tilde{Y}^{(1)})^{M'} - (\tilde{Y}^{(2)})^{M'})(\mathbf{1}^{k+1})^{M'}\right] > \mathbb{E}\left[((\tilde{Y}^{(1)})^M - (\tilde{Y}^{(2)})^M)(\mathbf{1}^{k+1})^M\right]$$

When bidder $k+1$ wins in the weak dominant strategy equilibrium of the second-price auction, then $(\tilde{Y}^{(1)})^{M'} = \tilde{Y}_{M'}^{k+1}$. In this case, we also have $(\tilde{Y}^{(2)})^{M'} = \max\{\tilde{Y}_{M'}^1, ..., \tilde{Y}_{M'}^k, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^n\}$. However, from an agent's perspective who aware of M, she knows that bidders 1 to k are aware of characteristic ℓ. Thus, $(\tilde{Y}^{(2)})^M = \max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^n\}$. Hence, above inequality can be rewritten as

$$\mathbb{E}\left[(\tilde{Y}_{M'}^{k+1} - \max\{\tilde{Y}_{M'}^1, ..., \tilde{Y}_{M'}^k, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^n\})(\mathbf{1}^{k+1})^{M'}\right]$$
$$> \mathbb{E}\left[(\tilde{Y}_{M'}^{k+1} - \max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^n\})(\mathbf{1}^{k+1})^M\right]$$

If $\mu_\ell^i > 0$ for $i = 1, ..., k$, then $\max\{\tilde{Y}_{M'}^1 + \tilde{X}_\ell^1, ..., \tilde{Y}_{M'}^k + \tilde{X}_\ell^k, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^n\} > \max\{\tilde{Y}_{M'}^1, ..., \tilde{Y}_{M'}^k, \tilde{Y}_{M'}^{k+2}, ..., \tilde{Y}_{M'}^n\}$ a.s. Moreover, an agent aware of M also realizes that the probability of $k+1$ winning is lower than k perceives if $\mu_\ell^i > 0$, $i = 1, ..., k$, because bidder $k+1$ does not realize that bidders 1 to k are aware of ℓ with $\mu_\ell^i > 0$, $i = 1, ..., k$. This proves the inequality. $\square$

Proof of Proposition 4

Suppose originally the bidders are aware and informed of the set of characteristics M' with $\ell \notin M'$. The seller's expected revenue is therefore $\mathbb{E}[(Y^{(1)})^M]$. By raising the common awareness of characteristic ℓ among all bidders, the seller's expected revenue becomes $\mathbb{E}[\max\{Y^1 + \mu_\ell, Y^2 + \mu_\ell, \ldots, Y^n + \mu_\ell\}] = \mathbb{E}[\max\{Y^1, Y^2, \ldots, Y^n\}] + \mu_\ell$, which is strictly greater than $\mathbb{E}[(Y^{(1)})^M]$ if and only if $\mu_\ell > 0$. $\square$

Proof of Proposition 5

Suppose originally bidders are informed on the set of characteristics M' where $\ell \notin M'$. With the finest information the seller's expected revenue is exactly $\mathbb{E}[(Y^{(1)})^M]$ which is equal to $\mathbb{E}[\max\{\sum_{j \in M'} X_j^1, \sum_{j \in M'} X_j^2, \ldots, \sum_{j \in M'} X_j^n\}]$. When all bidders are aware of the set of characteristics, $M' \cup \{\ell\}$, the best the seller can obtain is $\mathbb{E}[\max\{\sum_{j \in M'} X_j^1 +$

$X_\ell^1, \sum_{j \in M'} X_j^2 + X_\ell^2, \ldots, \sum_{j \in M'} X_j^n + X_\ell^n\}].$

$$\max\{\sum_{j \in M'} X_j^1 + X_\ell^1, \sum_{j \in M'} X_j^2 + X_\ell^2, \ldots, \sum_{j \in M'} X_j^n + X_\ell^n\}$$

$$\leq \max\{\sum_{j \in M'} X_j^1, \sum_{j \in M'} X_j^2, \ldots, \sum_{j \in M'} X_j^n\} + \max\{X_\ell^1, X_\ell^2, \ldots, X_\ell^n\}$$

where the inequality follows from the convexity of the maximum. Then with the expectation operator and due to the Jensen's inequality and the convexity of the maximum,

$$\mathbb{E}\left[\max\{\sum_{j \in M'} X_j^1 + X_\ell^1, \sum_{j \in M'} X_j^2 + X_\ell^2, \ldots, \sum_{j \in M'} X_j^n + X_\ell^n\}\right]$$

$$\leq \mathbb{E}\left[\max\{\sum_{j \in M'} X_j^1, \sum_{j \in M'} X_j^2, \ldots, \sum_{j \in M'} X_j^n\}\right] + \mathbb{E}\left[\max\{X_\ell^1, X_\ell^2, \ldots, X_\ell^n\}\right]$$

$$< \mathbb{E}\left[\max\{\sum_{j \in M'} X_j^1, \sum_{j \in M'} X_j^2, \ldots, \sum_{j \in M'} X_j^n\}\right]$$

if and only if $\mathbb{E}[\max\{X_\ell^1, X_\ell^2, \ldots, X_\ell^n\}] < 0.$ $\qquad\square$

Proof of Proposition 6

If all bidders are aware of the same set of characteristics, following Corollary 1 the seller's expected revenue is equal to $\mathbb{E}[(\tilde{Y}^{(1)})^M]$. By Jensen's inequality and the convexity of the maximum, it follows that $\mathbb{E}[(Y^{(1)})^M] \geq \mathbb{E}[(\tilde{Y}^{(1)})^M]$. $\qquad\square$

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