Value Capture in the Face of Known and Unknown Unknowns*

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Abstract

A large theoretical literature on value capture following Brandenburger and Stuart (1996) uses cooperative games under complete information to study how and why firms earn supernormal profits. However, firms often have different information, beliefs, or creative foresight. We extend value capture theory to incomplete information ("known unknowns") or unawareness ("unknown unknowns"), and illustrate some conceptual issues with that extension. Using the case study of Cirque du Soleil, we show how an entrepreneurial firm can profit even when it does not contribute materially to value creation. In a case study of Apple iTunes, we show how value capture depends quantitatively on the beliefs of other firms.

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"Reports that say that something hasn't happened are always interesting to me, because as we know, there are known knowns; there are things we know we know. We also know there are known unknowns; that is to say we know there are some things we do not know. But there are also unknown unknowns, the ones we don't know we don't know. And if one looks throughout the history of our country and other free countries, it is the latter category that tend to be the difficult ones."

Donald Rumsfeld, United States Secretary of Defense, February 12, 2002

1 Introduction

With their pioneering paper, Brandenburger and Stuart Jr. (1996) seeded what has now grown into a salient category of scholarship in strategy, often referred to as the "value capture" stream. An identifying feature of work in this area is its incorporation of quantitative tools supplied by cooperative game theory. The value capture stream has generated many important insights, from theoretical to empirical and from academic to practical. One of the ongoing goals of theorists working in this area is to develop a unifying, formal theory of strategy – that is, an analytical, explanatory framework that applies to a broadly inclusive domain of strategy phenomena. In this paper, we make an important, two-fold contribution to that goal and discover several strategy-relevant insights along the way.

There have been three major conceptual advances in value capture theory, each of which also extended the methodology toward broader ranges of strategy phenomena and deeper explanatory potentials. Brandenburger and Stuart Jr. (1996) presents the first advances in value capture theory in strategy, thereby establishing an entirely new stream of work. That paper's conceptual contributions to strategy research include the value-creation/value-capture distinction, the notion of an agent's added value, and the implication that an agent's added value is an upper bound to the value that agent can capture. Although Brandenburger and Stuart Jr. (1996) do not present a formal model, cooperative game theory and the core solution concept did inform their analysis. MacDonald and Ryall (2004) explicitly introduce this formalism to strategy. Using this formalism in conjunction with the core, they go beyond the analysis of Brandenburger and Stuart Jr. (1996) by showing that competition, in addition to its well-known corrosive effects on performance, can also work in the opposite direction - to guarantee an agent *positive* value capture. Finally, Brandenburger and Stuart Jr. (2007) innovates the "bi-form" game, which uses the standard cooperative game as the payoff-generating back-end to a non-cooperative game front-end. This setup allows assessment of the consequences of firms' strategic moves on their performance and broadens the analytical domain to dynamic settings.

Together, the methodological and conceptual contributions of these papers create a nice foundation for what is now a substantial stream of work, replete with further insights and novel findings.[1] While this is all for the good, value capture theory still falls short of its goal of presenting a broadly unifying analytical framework for strategy research. To be sure, there has been progress. For example, industry positioning (e.g., Porter, 1980), the resource-based view (e.g., Wernerfelt, 1984; Barney, 1991), and organizational network theory (e.g., Burt, 2009)

[1] For a recent survey, see Gans and Ryall (2017).

have all received explicit treatment using value capture analysis.[2] Nevertheless, some (like Ross, 2018) suggest that many important strategy issues remain open because the existing formalism has been underutilized. Others (like Menon, 2018)) argue that the existing formalism, based as it is upon the use of complete information cooperative games, is – in principle – unable to treat a number of phenomena of central interest to strategy scholars.

What is the problem? Central to value capture theory are cooperative games with *complete information*. In these games, potential for value capture is summarized by a characteristic function. This function states the economic surplus each subset of market agents can produce among themselves. The central solution concept is the *core*. It provides bounds on profit implied by competition given the characteristic function values. However, complete information means that these bounds rely upon the implicit premise that agents share expectations about the actual aggregate value that will be created by the market as well as the values that could be created by all possible combinations of agents were they to transact among themselves.[3] The problem is that these strong information requirements are precisely in conflict with strategy phenomena that are *driven by* differences in the belief systems, cognitional limitations, and mental models of the agents involved.

Consider, for example, the conjecture by Barney (1986) that different expectations can result in above-average performance for some firms and the suggestion by Lippman and Rumelt (1982) that uncertain imitability can sustain them.[4] Similarly, Schilling (2018) points out that "visionary" strategies – surprising moves that change the course of an entire industry – may owe their remarkable effects to the fact that competitors do not share the perceptions of the visionary strategist.[5] An important line of work in strategy centers on heterogeneous imperfections in the cognitive representations used by agents to determine their actions (e.g., Gavetti and Levinthal, 2000; Levinthal, 2011). More generally, that agents may simply be *unaware* of the options available to them and others is, if not explicitly then implicitly, central to those streams of strategy scholarship in which the focus is upon knowledge creation and diffusion.[6] Unawareness may include well-understood constituents that happen to be organized in a novel way.[7] Even practitioner-popular theories of strategy, like "blue ocean strategy" (Mauborgne and Kim, 2005), rely upon the existence of substantial inconsistencies in worldviews across market participants. What all of these examples share, contra the complete information assumption, is the view that

[2] See, e.g., Brandenburger and Stuart Jr. (1996), Chatain (2011), and Ryall and Sorenson (2007), respectively.

[3] It is worth pointing out that although the approach features complete information, it does *not* necessarily imply that it could not be applied to settings with imperfect information. Rather, the assumption is that, all agents have *symmetric* information about the value and it is commonly known that all agents have symmetric information about the value (see also Section 5.3).

[4] In their paper on the use of cooperative game theory in strategy research, Lippman and Rumelt (2003, p. 1084) return to this idea, speculating that real markets entail "a vast set of games" generated by the different resource uses discovered by competing firms as a result of their uncertain searches.

[5] This is closely related to ideas in the early literature on entrepreneurship, which portrays successful innovators as those with unique imaginative foresight, superior awareness, and alertness to market opportunities (e.g., Schumpeter (1911), Knight (1921), Kirzner (1973)).

[6] Durand et al. (2017) describe the breadth of these inquiries in their review.

[7] This idea is expressed quite eloquently by (Drucker, 1985, p. 135), who writes "Indeed, the greatest praise an innovation can receive is for people to say: 'This is obvious. Why didn't I think of it?'"

disagreement about the value a group could create – or even the very possibility of creating any value at all – is at the heart of the matter.

Our primary contribution is to extend the general apparatus of value capture theory to permit the modeling of situations like those discussed above. These are settings in which market participants disagree about the basics of their joint, value-creating opportunities (i.e., as summarized by a characteristic function). We first extend the standard model to allow for incomplete information (i.e., "known unknowns" in Rumsfeld's terminology) regarding the characteristic function. These are situations in which agents are aware of relevant events that affect amounts of producible value, but may be uncertain about the events' occurrences, and may even disagree on the probabilities with which those events occur. For example, consider Apple's iTunes. Apple captured extraordinary value when partnering with major music businesses to add content to its iTunes Store and high margin hardware devices. As we will show, this success depended on the belief of the music industry that online music retail was likely to be a low-profit business. Our model can also represent agents who are unaware of some possibility, rather than just thinking it unlikely to occur. This formalism allows us to model an example like Cirque du Soleil, in which some entrepreneurs innovated a novel form of live entertainment as an unexpected synthesis of preexisting categories.

Our technical challenge is to overcome the impossibility of modeling unawareness with standard state-spaces, as shown by Dekel et al. (1998). To wit, we marry value capture theory to recent developments on mathematical modeling of incomplete information and unawareness.[8] We first devise characteristic function games with incomplete information and extend the *coarse core* (Wilson (1978)) to those games. In a second step, we develop characteristic function games with incomplete information *and* unawareness by applying unawareness structures developed in Heifetz et al. (2006), Heifetz et al. (2008), and Heifetz et al. (2013a), and extend the coarse core to them. All of these extensions are novel.[9]

We start in Section 2 with a discussion of the Cirque du Soleil case (Casadesus-Masanell and Aucoin, 2009). This case allows us to illustrate our formal apparatus in a gentle way. It demonstrates the relevance and applicability of our approach with a particular business case. Finally, it enables us to discuss the difference between unawareness and incomplete information.

In Sections 3 and 4, we show how to formally extend cooperative games to settings with incomplete information and unawareness, respectively. We allow the characteristic function to be state-dependent. Thus, value creation can depend on a wide variety of events, including: agents' willingness to pay for a given product, suppliers' costs, the products rivals could produce if they formed a venture, and so on. States also describe beliefs and awareness of those events for each agent, including any higher-order beliefs about beliefs and awareness. This allows agents to

[8] In the current paper we focus both on incomplete information, formalized with probabilistic beliefs, and unawareness. It is possible to extend our framework to allow for ambiguity and Knightian uncertainty, in which groups may also lack confidence in forming probabilistic judgments. While we find ambiguity highly relevant in the context of strategy (see for example Ryall, 2009; Ryall and Sampson, 2017), we reserve that extension for a later paper.

[9] In economics, the closest recent literature is on contracting using noncooperative games with unawareness. von Thadden and Zhao (2012) and Auster (2013) study principal-agent problems with moral hazard. Auster and Pavoni (2019) and Lei and Zhao (2019) consider optimal delegation under unawareness. Francetich and Schipper (2020) analyze screening under unawareness. All these papers focus on particular contracting situations while we are interested in relatively detail-free, multi-lateral, free-forms of value creation and value capture.

have asymmetric expectations about the consequences of the actual deals they intend as well as about the alternative deals that are not intended but which, nevertheless, could be implemented.

Value capture in this model is, as in earlier research assuming complete information, constrained by competition. The captured values must be feasible in the sense that a distribution in the core cannot distribute more value than the aggregate amount created among all the agents. In addition, the captured values must be consistent with competition in the sense that no group of agents would want to deviate from them and pursue alternative side-deals instead. With incomplete information or unawareness, competitive consistency is a subtle idea. It requires that no group of agents can *agree among each other* to deviate from the proposed distribution of value. Here agreement means that there is explicit common knowledge among group members, which is very much in the spirit of the famous no-agreeing-to-disagree theorem in Aumann (1976).[10] Our solution concept is based on the coarse core originally introduced by Wilson (1978) for exchange economies with asymmetric information, extended to characteristic function games with incomplete information and unawareness.

While our paper remains largely expository in nature, we do illustrate the usefulness of our framework with some general insights for strategy. It is well-known from Brandenburger and Stuart Jr. (1996) that under complete information added-value places an upper bound on profits captured by a firm. Our analyses of the Cirque du Soleil and Apple iTunes cases show that this does not need to be the case under incomplete information and unawareness. In both cases, a firm can earn profits substantially beyond its added value. Nevertheless, we prove a generalization of Brandenburger and Stuart Jr. (1996) according to which it cannot be common certainty that a firm earns expected profits strictly above its expected added value. Next, from MacDonald and Ryall (2004), it is known that in order for an agent to capture positive profits, it is necessary that she adds strictly positive value. Again, our analysis of the Cirque du Soleil case reveals that this does not need to be the case under incomplete information and unawareness. Yet, we prove a generalization of MacDonald and Ryall (2004) according to which it cannot be common certainty that everybody expects a firm to have zero expected added value and this firm to obtain strictly positive expected profit.

Finally, in Section 5 we discuss features and limitations of our current approach, and avenues to address the latter in future research. In particular, we argue that strategic disclosure of information needs to be explicitly modeled if we are to capture certain empirically relevant details of information and awareness. This is left for future research.

2 Illustrative Example: Cirque du Soleil

As an example of a successful strategy of the "blue ocean" kind that they admonish managers to pursue, Mauborgne and Kim (2005) present the case of Cirque du Soleil.[11] Cirque implemented a fresh take on the traditional circus – an ingenious synthesis of existing categories of live performance into a novel category of entertainment. Important features of this synthesis included: keeping the cachet of a circus "big-top" tent; dropping live animal acts; using street performers

[10] While perfect information models feature common knowledge in an informal way, the fact that we now allow agents to disagree about value capture means we must make this requirement explicit.

[11] Casadesus-Masanell and Aucoin (2009) provide a detailed business case study.

rather than star acrobats; and imbuing the performance with a sophisticated, dramatic narrative played out to an original soundtrack. This insight led Cirque's revenue to increase twenty-fold between 1995 and 2007.

Why did Cirque earn so much profit? We suggest, following previous qualitative research, that this synthetic vision held uniquely by Cirque and not by other circus troupes led to a level of profit that would not have occurred if the same vision was widespread. Since Schumpeter (1911), innovation has been characterized as a process of recombination of known components.[12] In some inventive activities, agents may be aware of all possible recombinations. They are simply uncertain which ones yield the desired success. The well-defined quest of research and development is then to find out the right recombination. This is a situation of incomplete information.

There are other settings in which some agents are unaware of the emergent consequences of some recombinations. It then takes creativity to discover these consequences. Indeed, very much in the Schumpeterian paradigm of innovation, the theory of creativity by Koestler (1964) characterizes the creative act as recombining frames of mind that would not be conventionally associated with each other. It appears that, prior to Cirque, no one foresaw the emergent value created by the combination of its components – elements of a traditional circus, street performance, drama, and a concert – even though they existed in plain sight for any market participant. From a strategy perspective, unawareness explains the central question of why no one came up with this winning synthesis earlier. The providers of traditional circus acts were unaware of the opportunity Cirque envisioned and then implemented, right under their noses.

To illustrate our approach for analyzing this important class of business strategy phenomena, let us consider a highly stylized version of the Cirque case. Consider three firms: Cirque (C), Street Performance Inc. (P), and Big Top Circus (B). Firms P and B have extensive experience operating in the street performance and big-top circus market segments, respectively. P and B are not aware of any opportunity to work together in some innovative fashion and C is inactive. To stick with the popular metaphor, we call this the "red ocean" state and denote it r. In state r, we assume P captures value of 10 and B captures value of 30 from independent operations.

Now, suppose that C has the idea of combining P and B into an altogether novel form of entertainment that is much more valuable than what P and B create independently. Assume that if the market is receptive to it, the value of such a novel form of entertainment is 100. Otherwise, if it flops, no more value is produced than under the traditional way of operating (40 in aggregate). P and B remain unaware of this possibility – even though the idea is simple, P and B do not have it. Importantly, we also assume that C is certain that P and B do not have it. Finally, although C is aware of the idea to recombine P and B in a novel way, it is uncertain as to the receptivity of this novel form of entertainment by the market. Specifically, assume it assigns probability $\frac{1}{2}$ to the recombination of P and B being a hit and $\frac{1}{2}$ to being a flop. We may call these the "blue ocean" states h and f, respectively.

Note that C brings *nothing* to the table (in terms of value production): P and B could create the \$100 without C if only they thought of it. Moreover, C does not know whether h or f is true. In the technical sense that an agent "knows" the event (subset of outcomes) to which it assigns probability equal to one, C knows nothing. Indeed, C is maximally uncertain as to

[12]This idea has been extended to strategy. For example, Lippman and Rumelt (2003) propose that a central strategic issue facing a firm is identifying the best combination of resources.

whether the true state of the world is that its idea is a hit or a flop. The only thing C has going for it is awareness of the possibility of combining P and B into a novel form of entertainment which, at this point, is just an idea.

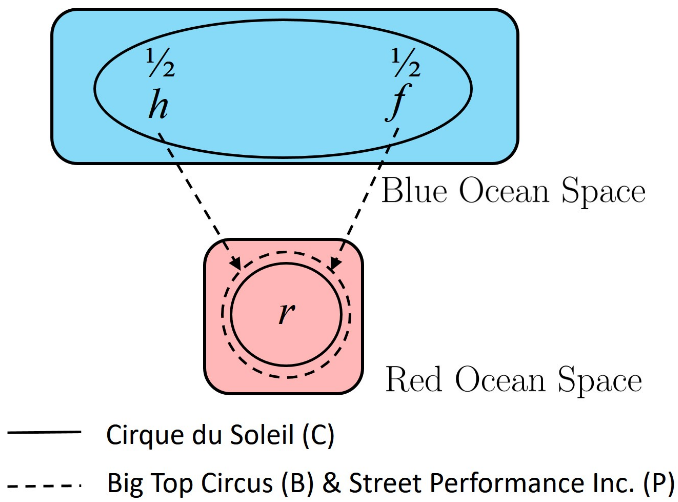

Figure 1: Unawareness Structure in the Cirque Example

How can these possibilities be modeled? As we will discuss in more depth in Section 4, standard state-space models in decision theory do not allow agents to be unaware of the possibility of some states. We therefore use the unawareness structures introduced by Heifetz et al. (2013a). The approach is illustrated for the Cirque example in Figure 1. There are *two* state spaces, the Blue Ocean Space and the Red Ocean Space. The Red Ocean Space (lower space) contains just one state, r, which represents P and B conducting independent operations in their traditional markets and C being inactive. Firms P and B can only reason about r, as shown by the dashed circle around it in the diagram. Circles and ovals indicate supports of probability distributions: P and B are certain of r.

Moreover, because they are unaware of any other possibility, they must also assume that everyone else also reasons only in the Red Ocean Space. Thus, P and B imagine the state of mind of C is indicated by the solid-lined circle in the Red Ocean Space. In reality, C is aware of its novel idea. This is represented by the Blue Ocean Space, where C's true state of mind is given by the solid-lined oval that contains both states h and f (with its beliefs shown above each state). Since at both h and f the beliefs of P and B are concentrated on r in the Red Ocean Space (as indicated by the dashed arrows emanating from both h and f and pointing to r), C is certain that both P and B are unaware of its novel idea.

Simple as it is, Figure 1 illustrates how unawareness structures go beyond standard state spaces. First, there are several state spaces, one for each awareness level. These spaces have a natural order by expressiveness. The Blue Ocean Space is more expressive than the Red Ocean Space: its states describe the success or failure of the innovative form of entertainment, which is not possible in Red Ocean Space. Second, an agent's belief at a state in some space may be concentrated on states in a less expressive space. For instance, at any state in the Blue Ocean Space, firms B and P's beliefs are on the Red Ocean Space. They have no beliefs about states in the Blue Ocean Space at all, which is how B and P's unawareness of the Blue Ocean is formalized in this structure.

Next, we explain how value creation works by introducing the state-dependent characteristic function shown in Table 1. The table shows, for each state (and space) and each group of firms, the quantity of value created by the associated group in the associated space. There are two items worth noting. First, in every state, C adds zero value to all groups. Second, the value created in state f is the same as that in state r. Thus, C does not know whether the innovative idea will lead to a hit or business as usual. In fact, since C holds a uniform belief over h and f, it is maximally uncertain about hit versus failure. Given that C is unable to contribute any productive capacity or information, it is intuitive to conjecture that C should also be unable to capture any value. Nevertheless, our intuition is that C *should* receive some payoff because, after all, the idea really is rare and valuable. In fact, the insight of the example will be to demonstrate that an agent can capture value by just contributing awareness instead of contributing any productive capacity or information.

What do values represent here? We will discuss this point in greater depth, but it is important to see both that the characteristic function is *state-dependent*, and that values in these state-dependent characteristic function represent the *actual value which will be created* in those states given agents' beliefs and unawareness. That is, when we say that the group $\{P, B, C\}$ has a value of 100 in state h, we mean that, from the perspective of the modeler, 100 would actually be created by that group if that state obtains. Recall that, in cooperative game theory, characteristic functions represent value created in free-form interaction between groups or subgroups. So, implicitly, h represents all the events and unmodeled actions of the agents that would lead to the creation of 100 worth of value were this state to occur.

Blue Ocean Space		
Groups	h	f
$\{P, B, C\}$	100	40
$\{P, B\}$	100	40
$\{P, C\}$	10	10
$\{B, C\}$	30	30
$\{P\}$	10	10
$\{B\}$	30	30
$\{C\}$	0	0

Red Ocean Space	
Groups	r
$\{P, B, C\}$	40
$\{P, B\}$	40
$\{P, C\}$	10
$\{B, C\}$	30
$\{P\}$	10
$\{B\}$	30
$\{C\}$	0

Table 1: Values of the State-Dependent Characteristic Function in the Cirque Example

What does this value creation imply about value capture? To answer this, let us illustrate our solution concept, the *coarse core*. Since there are just three agents we can derive the coarse core geometrically using simplices, one for each state.[13] Requiring a state-dependent distribution of captured values to be contained in that state's simplex amounts to imposing feasibility: at every state, the distribution of value cannot exceed the value created at that state. To impose competitive consistency, we indicate in each simplex linear constraints imposed by alternative, value-creating opportunities available to the various groups (including by each single firm).

[13] A simplex shows all ways of distributing the aggregate value associated with that state.

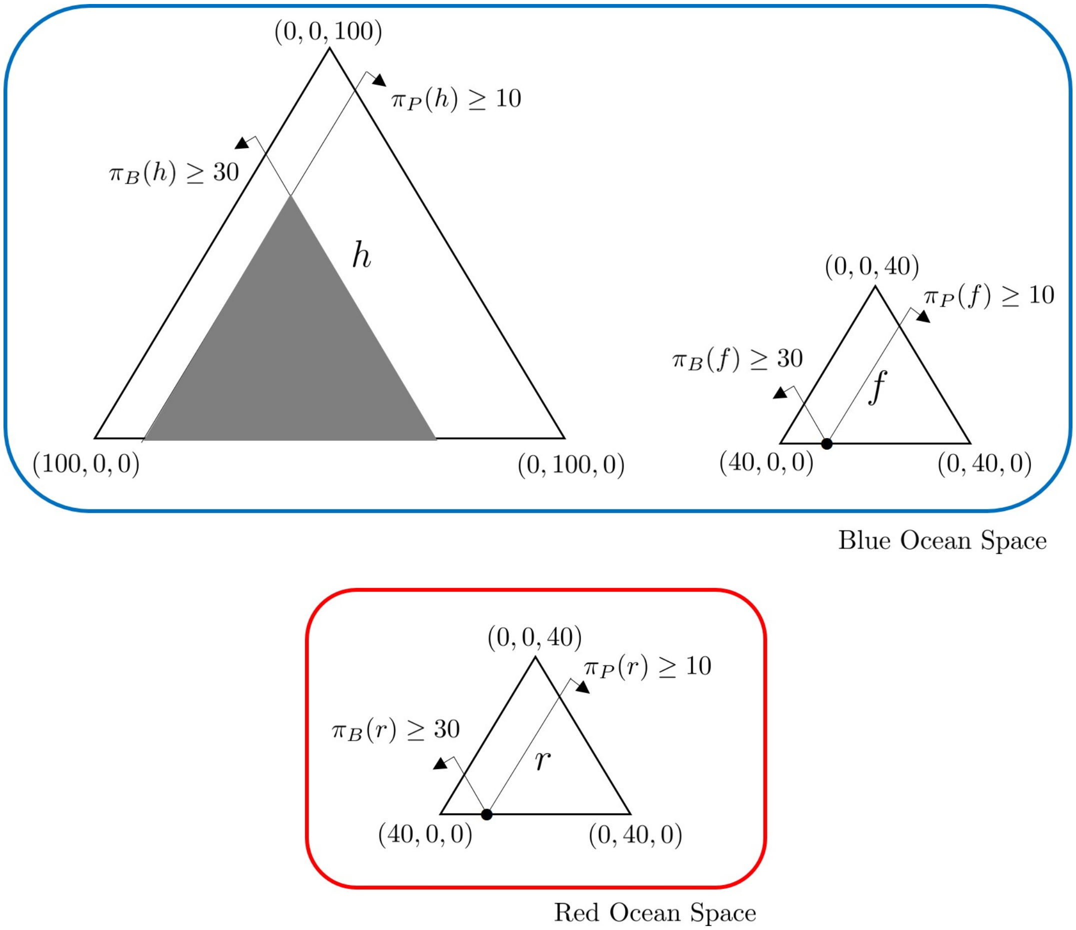

Figure 2: Geometry of the Coarse Core in the Cirque Example

Here, it is sufficient to examine the constraints associated with B and P, the only agents with any alternatives to C's innovative scheme. Consider the diagram presented in Figure 2. There are two simplices, one for each state of the Blue Ocean Space, as well as one simplex for the state in the Red Ocean Space. The size of each simplex is drawn to scale, indicating the relative amount of value created in aggregate by all firms in that state. Each point in a simplex represents a feasible distribution of value. For instance, on the upper-left "big" simplex corresponding to state h of the Blue Ocean Space, the extreme point in the lower left corner corresponds to the distribution of value (0, 100, 0), where the number on the left represents B's value capture (zero), the middle number is P's capture (100), and the number on the right is the value captured by C (zero).

The diagram shows that, because the aggregate quantities of value created vary across states, the distribution of that value varies as well. For example, the maximum a firm can conceivably capture in state h is 100. In state f in the Blue Ocean Space and state r in the Red Ocean Space, that maximum is only 40. We denote by $\pi_B(h)$ the value captured by B in state h, by $\pi_P(f)$ the value captured by P in state f, and so on. The subscript refers to the firm. The argument refers to the state. With this notation we can discuss the coarse core.

Begin with the Red Ocean Space. At state r, all firms are unaware of the Blue Ocean (according to the unawareness structure in Figure 1). B knows she can capture value of 30 on her own, P knows she can capture 10 on her own, and C cannot capture anything independently (as shown in Table 1). In r, aggregate value capture cannot exceed 40. At the same time, given their independent alternatives in this state, it must be that $\pi_B(r) \geq 30$, $\pi_P(r) \geq 10$ and $\pi_C(r) \geq 0$. The linear constraints corresponding to the independent alternatives of B and P are shown in the diagram as lines in the simplex corresponding to state r.[14] The only distribution of value satisfying these conditions is $\pi_B(r) = 30$, $\pi_P(r) = 10$, and $\pi_C(r) = 0$ (as indicated by the dot in the simplex corresponding to state r). In the Red Ocean world of state r, these are values that would be captured by the three firms.

Next, consider the Blue Ocean states. According to the unawareness structure in Figure 1, B and P are unaware of the Blue Ocean at any state in the Blue Ocean Space. C is aware of the Blue Ocean at either h or f. C is also certain that B and P are not aware of the Blue Ocean. Thus, when C considers putting together a deal involving B and P, it realizes that the relevant alternatives, from the perspectives of B and P, are the ones associated with the Red Ocean Space. Specifically, at any state of the Blue Ocean Space, B must capture at least 30 and P at least 10 because these are the amounts of value they know they can obtain independent of C. Again, these are linear constraints, easily depicted as lines shown in the simplices of the Blue Ocean Space.

Summing up, any state-contingent distribution of value in the coarse core must respect all of these constraints. Specifically, any distribution (π_B, π_P, π_C) satisfying

$$100 \geq \pi_B(h), \pi_B(f), \pi_B(r) \geq 30$$

$$100 \geq \pi_P(h), \pi_P(f), \pi_P(r) \geq 10$$

$$100 \geq \pi_C(h), \pi_C(f), \pi_C(r) \geq 0$$

$$\pi_B(h) + \pi_P(h) + \pi_C(h) \leq 100$$

$$\pi_B(f) + \pi_P(f) + \pi_C(f) \leq 40$$

$$\pi_B(r) + \pi_P(r) + \pi_C(r) \leq 40$$

is in the coarse core. At state h of the Blue Ocean Space, any distribution corresponding to a point in the grey triangle is in the coarse core. At states f of the Blue Ocean and r of the Red Ocean, the distribution of payoffs in the coarse core corresponds to the black dot in which firm B captures 30, firm P 10, and firm C nothing.

Consistent with our earlier intuition, in state h there is nothing preventing C from capturing a substantial amount of value – even though it does not contribute anything to the value created and is maximally uncertain of the true state. What C *does* contribute is imaginative foresight, her awareness of a novel business idea which creates the Blue Ocean. For instance, the distribution $(\pi_B(h), \pi_P(h), \pi_C(h)) = (30, 10, 60)$ is in the coarse core, in which both B and P's constraints are binding and C captures the entire residual. Even so, C is not certain whether state h or f obtains, assigning probability $\frac{1}{2}$ to both states. Thus, if C captures all the residual value when the outcome is a hit, her *expected* value capture is 30, which is still substantial.[15]

[14] The constraint corresponding to C is redundant and, hence, not depicted to avoid clutter.

[15] Since $(\pi_B(h), \pi_P(h), \pi_C(h)) = (30, 10, 60)$ and $(\pi_B(f), \pi_P(f), \pi_C(f)) = (30, 10, 0)$.

We call this an *awareness rent*, i.e., a rent due to higher awareness or entrepreneurial foresight. Without C's awareness, no one would have thought to innovate in the maximally productive way.

How would these distributions be implemented? Under these conditions, we imagine that C could simply propose bilateral employment contracts to B and P, guaranteeing them wages equal to \$30 and \$10, respectively. Essentially, the emergence of bilateral employment contracts is what happened in the real-life Cirque du Soleil case. Of course, bilateral contracts with non-compete clauses, etc., may also help to sustain firm C's supernormal value capture by creating legal barriers to B and P cutting C out of the picture once B and P have the "why didn't we think of this" response. This illustrates that, in a world of unawareness, other legal institutions besides those that protect intellectual property may facilitate value capture. Alternatively, C may develop crucial knowledge with respect to producing and marketing these shows, such that it continues to enjoy positive value capture into the future.[16]

As mentioned in the Introduction, this discussion immediately points toward those lines of strategy research that focus on knowledge-based advantage. Sustainable value capture advantage may well begin with strategic insight, which then establishes first-mover advantages due to learning. As Drucker (1985) remarks, market incongruities that yield new business opportunities are often overlooked by insiders, since "(t)ypically, these incongruities are macro-phenomena, which occur within a whole industry or a whole service sector. The major opportunities for innovation exist, however, normally for the small and highly focused new enterprise, new process, or new service. And usually the innovator who exploits this incongruity can count on being left alone for a long time before the existing businesses or suppliers wake up to the fact that they have new and dangerous competition." Being left alone for a while may be enough to build know-how and exploit learning curves in ways that assure ongoing value capture. These dynamic considerations are beyond the scope of the present paper; our hope is that the model presented here motivates future research that goes beyond our static setting (see our discussion in Section 5).

At this point, it may be useful to contrast unawareness or unknown unknowns with incomplete information or known unknowns. We can use the Cirque du Soleil case to ask what would happen under incomplete information but common awareness? Suppose the agents do have asymmetric information. That is, consider a situation suggested by Barney (1986), in which different market participants have different beliefs with respect to the value of their resources. This situation can be represented by an incomplete information model. Specifically, assume as before that there is a state "hit," h, and a state "failure," f, and that C continues to believe that h and f each occur probability $\frac{1}{2}$. Unlike the case with unawareness, we assume now that P and B are aware of the novel combination but disagree with C about the likelihood of h and f. They each believe h is very unlikely, placing only probability $\frac{1}{10}$ on that outcome. The information structure is illustrated in Figure 3. There is just one state space containing two states, h and f. The blue solid-lined information set and probabilities belong to C. The red dashed information and red probabilities indicate the beliefs of both P and B.

The state-contingent characteristic function determining the values of all groups is given as

[16] See, for example, the recent analysis by Gans and Stern (2017) that analyzes entrepreneurial value capture through formal intellectual property protection (which they refer to as a "control" approach) versus through learning advantages (which they refer to as an "execution" approach).

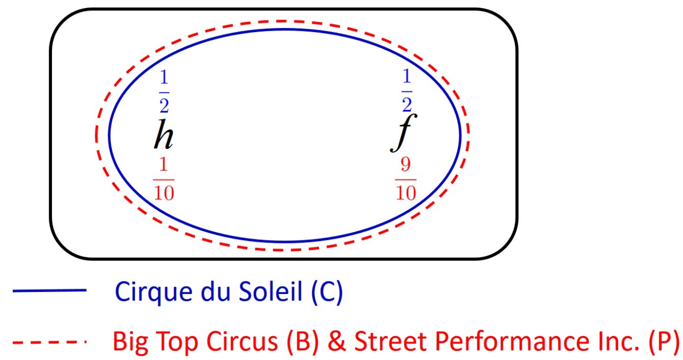

Figure 3: Information Structure in the Case of Incomplete Information

before for the Blue Ocean Space in Table 1. The difference now, which will turn out crucial for value capture, is that both P and B are aware of the value that they generate together in state h. That is, both P and B are now fully aware that together (and without agent C) they can generate a value of 100 in state h.

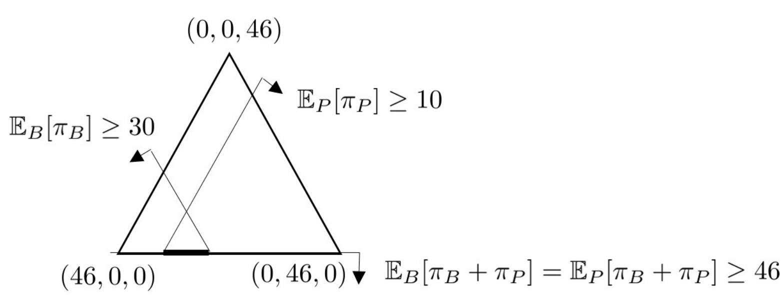

Figure 4: Geometry of the Coarse Core in the Case of Incomplete Information only

What bounds does the coarse core impose on value capture in this example under incomplete information but common awareness? Clearly, in each state, no more can be allocated than what is generated. Thus, as before, at state h no more than 100 can be distributed whereas at state f no more than 40 can be distributed. Next, B is certain of the independent possibility of earning 30 in each state, while P is similarly certain of earning at least 10 in each state. Thus, B will block any state-dependent allocations that do not yield an expected profit of at least 30. Similarly, P blocks any allocations that yield an expected profit of less than 10. Together, they agree they can earn an expected profit of $\frac{1}{10}100 + \frac{9}{10}40 = 46$. Thus, together they will block any allocations that earns them less than 46 in expectation. At the same time, they do not expect to earn more than 46. Therefore, the size of each side of their "expected simplex" is just 46. This simplex and the constraints are depicted in Figure 4.[17]

[17] There is also an "expected simplex" for C whose sides have length 70. Yet, no relevant constraints are imposed by C in the coarse core. Note that although C's expected value created by group $\{P, B, C\}$ exceeds the value of 46 expected by P and B, it cannot offer them an expected profit larger than what they could achieve with any allocation satisfying the constraints below since this would violate state-wise feasibility and the assumption of no losses discussed in Section 3.3.

We see that the constraints of the coarse core pin down a set of profit allocations in the segment indicated by the thick black line at the bottom of the triangle. Notice that the feasibility constraints combined with the requirement that B and P must earn 46 in expectation imply that B and P must, jointly, capture exactly 100 at h and 40 at f. It immediately follows that this holds if and only if C earns zero at both states – in stark contrast to the case with unawareness. The coarse core in the case of incomplete information is characterized by the following four constraints:

$$\pi_B(h) + \pi_P(h) = 100$$

$$\pi_B(f) + \pi_P(f) = 40$$

$$\frac{1}{10}\pi_B(h) + \frac{9}{10}\pi_B(f) \geq 30$$

$$\frac{1}{10}\pi_P(h) + \frac{9}{10}\pi_P(f) \geq 10$$

For example, a state-contingent distribution satisfying these conditions is: $\pi_B(h) = 90$, $\pi_P(h) = 10$, $\pi_C(h) = 0$; and $\pi_B(f) = 30$, $\pi_P(f) = 10$, $\pi_C(f) = 0$. This could be implemented through an employment contract in which agent B employs P and pays P its outside option of 10, whether or not the joint project is a hit.

The crucial point is that the incomplete information framework does not offer the possibility of strategic moves of the kind for which the Cirque case is commonly used to exemplify. The Cirque innovation and the success that followed could *not* have been the result of exploiting heterogeneous beliefs about the probability of success of an untested idea. Rather, the story is that, prior to Cirque, no one had the slightest inkling of its inventive business model. And this was true, even though all of its novel components – elements of a traditional circus, street performance, drama, and a concert – existed in plain sight for any market participant. Cirque represents a real-world phenomenon that demands explicit recognition of bounded perceptions; e.g., that some firms are visionary (Schilling, 2018), that managers have different mental models (Levinthal, 2011), or that unawareness is present (Menon, 2018).

Note that we are not claiming that asymmetric information is never a source of value capture! In the following section, we will give an example based on Apple's iTunes with precisely that feature, and explain how it differs from Cirque. Before moving to that example, however, let us fully lay out the general theoretical framework.

3 Value Capture under Known Unknowns

In this section we focus on extending value capture theory to incomplete information, in which agents are aware of all the states, but may have asymmetric beliefs about them. The extension to unawareness is left to the next section. Focusing on incomplete information first allows us to introduce the modeling complexities step-by-step.

3.1 Incomplete Information

Incomplete information refers to situations with known unknowns. Begin with an indexed set of market participants, denoted $N \equiv \{1, ..., n\}$, where n is finite. A generic participant, or *agent*, is denoted $i \in N$. To represent events of relevance to the market participants, we introduce a finite *state space*, denoted S, with typical element $\omega \in S$.[18] Each element in S includes a description of events that affect the economic values that agents can create by transacting with one another. An *event* is a subset of states, $E \subseteq S$. For instance, E could collect all states in which agent i has available some particular production technology. Let $\Sigma \equiv 2^S$ denote the set of all events; i.e., all subsets of S. We assume that agents are able to form probabilistic beliefs. To this end, let $\Delta(S)$ denote the set of all probability distributions over states in S, with typical element $\mu \in \Delta(S)$.

Unfortunately, just specifying a probability distribution over states is not enough for modeling beliefs of agents in strategic settings. The reason is that agents also need to form beliefs about other agents' beliefs, beliefs about beliefs about beliefs, and so on. The standard approach for this in game theory is the use of type spaces (Harsanyi (1967), Mertens and Zamir (1985)), in which a state encodes not only the "brute facts" that affect value creation, but also the beliefs of all the agents about those facts and about each others' beliefs. Specifically, for every agent $i \in N$, a *type mapping* is a function from states to probability distributions over states, $t_i : S \longrightarrow \Delta(S)$, where $t_i(\omega)$ represents agent i's beliefs over states when the state is ω. In this way, a state implicitly describes the beliefs of all the agents. The type mapping "pulls beliefs out of the state" to make them explicit. We impose the condition that, while agents can be uncertain about others' beliefs, they are always certain of their own.[19] This condition is called *Introspection*.[20]

3.2 Value Creation under Incomplete Information

The next step is to model value creation. As is standard in value capture theory, agents in a market are assumed to face myriad opportunities for the joint production of economic value, via simple arm's length transactions or more complex activities organized under elaborate contracts. As is the case in cooperative games with complete information, the value creation potential of arbitrary groups of agents is summarized by a characteristic function. However, in order to allow for incomplete information, we assume that the characteristic function is state-dependent. Since states describe not only the direct determinants of value creation but also the beliefs of market participants, agents may now disagree about the quantity of value producible by the various groups.

Let $\mathcal{G} \equiv \{G \subseteq N \mid G \neq \emptyset\}$ denote the set of all nonempty groups of agents. The *characteristic*

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[18]To avoid mathematical technicalities that add little additional insight, we take S to be finite. Our approach extends to infinite state spaces under appropriate measurability assumptions.

[19]This condition rules out some particular mistakes in information processing (e.g., Geanakoplos (2020), Samet (1990)). Formally, for each agent $i \in N$ and state $\omega \in S$, the agent's belief at ω, $t_i(\omega)$ assigns probability 1 to the set of states in which i has these beliefs: $t_i(\omega)(\{\omega' \in S \mid t_i(\omega') = t_i(\omega)\}) = 1$. In other words, the probability distribution representing an agent's beliefs is allowed to vary across states. However, at any given state, the agent's belief assigns probability 1 to the set of states where he has the same belief as in that state. Obviously, if beliefs are constant across states, introspection is satisfied.

[20]Note that this formalism allows the modeling of beliefs about another agent's beliefs, beliefs about those beliefs about beliefs, etc. Harsanyi's suggestion that such a representation can model arbitrarily higher order beliefs of agents was subsequently shown to be true by Mertens and Zamir (1985) and others under various assumptions about S.

function $v : S \times \mathcal{G} \longrightarrow \mathbb{R}_+$ assigns to each state $\omega \in S$ and each group of agents $G \in \mathcal{G}$ a nonnegative value $v(\omega, G) \geq 0$. We interpret $v(\omega, G)$ as the ex-post value that is created by group G when the true state is ω. By implication, $v(\omega, N)$ is the aggregate value that *is* produced at ω by the entire market.[21] Since, as we have seen above, states also pin down interim beliefs of agents, implicitly ex-post values may even depend on interim beliefs of agents. This is intuitive, as interim beliefs of agents may affect deal-making, production decisions, etc., of agents – features that, while not explicitly modeled in cooperative games, are implicitly picked up by the characteristic function.

We restrict the values of the characteristic function to non-negative real numbers. Under complete information, this is fairly innocuous since agents can and, presumably, would choose to avoid transactions known to result in losses. However, under incomplete information, a venture may result in substantial profits in some states and substantial losses in others. Naturally, groups may still decide to take calculated risks and join a venture when its expected value is sufficiently positive. Nevertheless, we rule out these situations to sidestep the issues of bankruptcy and default. We return to this point when discussing our solution concept in the next subsection and when we discuss limitations of our current model in Section 5.

The characteristic function approach has both its advantages and disadvantages. Rather than elaborating a framework to explicate how exogenous events and strategic actions by agents with various beliefs give rise to the actual deals that groups would strike to create value, we skip directly to the last step. That is, we assume all the details relevant to value creation are encoded in an abstract "state" and then take the state-contingent characteristic function as given. This allows for a detail-free approach to modeling the productive potentials of groups and, as such, greater generalization than that possible under a more specialized structure, such as a contract-theoretic (e.g., Bolton and Dewatripont (2005)) or industrial organization approach. Even so, this detail-freeness does introduce ambiguities about how to build a characteristic function from lower-level primitives. For example, the timing of actions in any deal-making process is an important determinant of the deals that get struck. We discuss this further in Section 5. For now we proceed under the assumption that, in each application, the interpretation of the characteristic function is fixed in a sensible way as we did in the Cirque du Soleil case of Section 2.

Taking stock of the framework we have introduced so far yields the notion of a characteristic function game with incomplete information. We summarize this formally in the following definition.

Definition 1 *A finite cooperative game with incomplete information* $\langle N, v, S, t_1, ..., t_n \rangle$ *consists of*

- *a finite set of agents,* N*;*
- *a finite state space* S*;*
- *for each agent* $i \in N$*, a type mapping* $t_i : S \longrightarrow \Delta(S)$ *satisfying Introspection; and*
- *a state-contingent characteristic function* $v : S \times \mathcal{G} \longrightarrow \mathbb{R}_+$*, where* $\mathcal{G} = \{G \subseteq N \mid G \neq \emptyset\}$*.*

[21] If this aggregate value is produced by a value network of agents in N (e.g., vertical supply chains or, more generally, "value partitions" as described by Montez et al., 2018), then these are also encoded by ω.

In a cooperative game with incomplete information, the value creation by groups of agents is state-dependent. In addition, each state determines a belief for each agent about states (and, thus, beliefs about the values created by various groups, beliefs about other agents' beliefs, etc.). Agents may have different opinions about values created. Note that, if S is a singleton (i.e., there exists only one state, which eliminates all uncertainty), then the preceding definition collapses to a standard cooperative game with complete information; i.e., in which each group is certain of the values created by various groups and this is common knowledge among all agents.

3.3 Value Capture under Incomplete Information

Let $\pi : S \longrightarrow \mathbb{R}_+^N$ be a function that assigns to each state $\omega \in S$ a distribution $\pi(\omega)$ of value. This distribution $\pi(\omega) = (\pi_1(\omega), ..., \pi_n(\omega))$ consists of a vector indicating quantities of value captured, one for each agent $i \in N$, where $\pi_i(\omega)$ is the value captured by agent i in the state ω. We now require a solution concept that ties the state-contingent value creation possibilities, as described by v, and the interim beliefs of the agents, as described by their types, to "sensible" distributions of value, as described by π.

The core has become the central solution concept in value capture theory because it has the nice interpretation of identifying the effects of competition on an agent's ability to capture value. Similar to the cooperative game itself, this concept is relatively detail-free, essentially capturing the outcomes of "free-form" bargaining among the agents. A distribution of value among agents is in the core if two properties are satisfied: First, the distribution of value among agents must be feasible: it cannot specify an aggregate level of value capture that is greater than the aggregate amount of value created in the market as a whole. Second, the distribution must be consistent with competition among agents: no group of agents should have an incentive to block the distribution due to a competing alternative to jointly create and share value among themselves. Put differently, there does not exist an alternative distribution of value that is feasible *among agents in the group* and that allows them to jointly capture more value than the amount specified in the original distribution.

It is not straightforward to define an analogous concept for cooperative games with incomplete information. Agents can have asymmetric beliefs about their joint opportunities. In these cases, how do we resolve when a distribution of value is consistent with these beliefs? Should the focus be on particular types of agents or all types? Is a distribution problematic when *all* agents of a group believe they have a strictly better alternative, or is it enough that *some* believe the alternative is strictly better and the rest are indifferent? Beyond this, is any information revealed – and, if so, by whom and to whom – during the implicit deal-making stage? Are some sort of updated, interim beliefs required? Can distributions of value depend on the private information of some agents and, if so, should incentives to reveal that information be checked? Different answers to these questions potentially lead to different solution concepts.

We now make several assumptions designed to mitigate these subtleties and specify a solution concept that satisfies them.

- First, we assume that any distribution of payoffs that constitutes a solution to the game must be *feasible by state*. That is, at any state, more value cannot be distributed than is created. Otherwise, since state-contingent characteristic functions represent ex-post values, it is not clear how such an infeasible distribution could be implemented.

- Second, we assume *no losses.* While we have already assumed that the characteristic function is non-negative, we also assume that individual levels of value capture are never negative. This is an uncontroversial assumption under complete information. No agent would agree to a venture in which she is sure to lose. However, it is not without loss of generality under incomplete information. For example, an agent might be willing to trade off losses in one state with substantial profits in another. By making this assumption, we avoid concerns about how to enforce losses ex-post, especially when it may potentially bankrupt some groups and lead to default.[22]

- Third, we assume that groups take their (interim) beliefs into account when comparing the value they capture against the productive opportunities facing them. Each agent uses the beliefs specified by her type when forming expectations about value created by various groups and the value captured in a particular distribution. This means that *no information is shared among agents* at the interim stage. An agent's implicit willingness or unwillingness to enter into alternative options does not convey additional information to other agents. Again, this assumption is not without loss of generality. One could consider settings in which the incentives of agents to share information at the interim stage is important. We avoid these complications here and leave them to a dynamic framework to be presented in future research.

Formally, we extend the coarse core due to Wilson (1978), who defined a core for the special setting of an exchange economy with asymmetric information and showed that, in this setting, it is nonempty (see also Kobayashi, 1980). We generalize the coarse core to cooperative games with incomplete information. This solution concept satisfies all the assumptions mention above, beginning with feasibility, which we formalize as follows:

Definition 2 (Feasibility) *Given a cooperative game with incomplete information* $\langle N, v, S, t_1, ..., t_n \rangle$, *a distribution of value* π *is* feasible for group G *if*

$$\sum_{i \in G} \pi_i(\omega) \leq v(\omega, G) \text{ for all } \omega \in S.$$

A distribution of value is feasible *if it is feasible for* N.

A distribution of value π is feasible for group G if, at *any* state the agents in G do not capture more value than they can create at that state. A special case is feasibility for all agents in N.

Next, we need to formalize a notion of competitive consistency. That is, we need to specify how competition constrains the set of feasible distributions of values captured. While under complete information, the values of the groups are commonly known, under incomplete information, agents may form (possibly different) expectations about the values created and values captured by various groups. These expectations constitute the basis for agents to agree or object to various distributions of value.

Given a distribution π and state $\omega \in S$, agent $i \in N$ forms expectations about the value she will capture by using her belief $t_i(\omega)$ at ω as follows:

$$\mathbb{E}\left[\pi_i \mid t_i(\omega)\right] := \sum_{\omega' \in S} \pi_i(\omega') \cdot t_i(\omega)(\{\omega'\}),$$

[22] This problem is analogous to bankruptcy in general equilibrium under incomplete information. See, for instance, Dubey et al. (2005).

where $t_i(\omega)(\{\omega'\})$ is the probability i assigns to ω' in state ω. We refer to $\mathbb{E}[\pi_i \mid t_i(\omega)]$ as *agent i's interim expected value capture at state $\omega \in S$.*

Intuitively, a distribution of value is consistent with the expectations of a group of agents if there are no feasible alternative distributions that imply better expected levels of value capture for the group. Thus, for any two distributions of value, π and π', we define *the event in which π' dominates π for agent i in expectation*:

$$\left\{\omega \in S : \mathbb{E}[\pi'_i \mid t_i(\omega)] > \mathbb{E}[\pi_i \mid t_i(\omega)]\right\}.$$

This is the set of states in which agent i expects to be strictly better off with the distribution of payoffs π' rather than π. Now, define *the event in which π' dominates π for group $G \in \mathcal{G}$*. This is simply the event that π' dominates π for $i \in G$ *and* for $j \in G$ *and* ... (for all agents in G):

$$\bigcap_{i \in G} \left\{\omega \in S : \mathbb{E}[\pi'_i \mid t_i(\omega)] > \mathbb{E}[\pi_i \mid t_i(\omega)]\right\}.$$

This is the set of states in which every agent in group G expects to be strictly better off with the distribution of payoffs π' than with π.

When would a group of agents G disagree with a proposed distribution of value? Suppose that agent i believes that he and agent j have a joint venture opportunity to create value and distribute it to themselves according to a distribution π' which, consistent with i's beliefs, dominates another distribution π for both i and j. Can π, therefore, be ruled out? Not necessarily. For instance, j might be uncertain about i's assessment of the joint opportunity. The fact that i proposes π' to j becomes now valuable information to j. Consequently j should update her beliefs based on the fact that i proposed π' to her. Yet, based on that update, maybe π' does not look that promising to j after all. Or perhaps there is now another distribution of value π'' that looks even more promising to j. Similarly, j accepting i's proposal for π' may be valuable information to i. All of this could result in a very complicated game of signaling and countersignaling, with alternative deals being proposed at each iteration.

This is an interesting issue, but one that is more properly addressed in a dynamic framework rather than in the static setting of this paper (see Section 5 for further discussions). Thus, we rule these sorts of machinations out by focusing on static interim beliefs. In particular, we assume that a distribution π is blocked by a group of agents G if it is common certainty among agents in G that some feasible π' dominates π for the group G. Common certainty in G here means that every agent in G assigns probability 1 to the event that π' dominates π for group G, every agent in G assigns probability 1 to the event that everybody in G assigns probability 1 to the event that π' dominates π for group G, and so on.[23]

Definition 3 (Blocking Group) *Given a cooperative game with incomplete information $\langle N, v, S, t_1, ..., t_n\rangle$, we say that a group of agents $G \in \mathcal{G}$ blocks a feasible distribution π at state $\omega \in S$ if there exists another distribution π' that is feasible for group G and it is common certainty among agents in G that π' dominates π for group G.*

We now have all the ingredients for our solution concept, the coarse core of Wilson (1978) extended to characteristic function games with incomplete information:

[23] In the appendix, we state the formal definition of common certainty.

Definition 4 (Coarse Core) *A distribution of value π is in the coarse core of the cooperative game with incomplete information $\langle N, v, S, t_1, ..., t_n \rangle$ if and only if*

(i) π is feasible, and

(ii) for all $\omega \in S$ no group blocks π.

A distribution of payoffs is in the coarse core if, at every state, it is feasible and there is no common certainty among any group of agents that some other feasible distribution for them results in them capturing strictly greater expected value.

We refer the interested reader to Appendix A, in which we provide a more rigorous exposition of this section with further details. We now illustrate our framework thus far introduced with the case of Apple iTunes Store.[24]

3.4 Example: Apple iTunes versus Music Industry

In 2003, two years after Apple introduced iTunes, a music management desktop computer program and device manager for its new iPod, it opened the iTunes Music Store. By 2008 it was the largest music vendor in the US, providing the content for Apple's high-margin hardware business. How did Apple take over the music market?

The music industry was under threat from digital downloading sites like Napster. Apple saw an opportunity for a legal alternative. It desperately needed content from the music industry to produce value alongside its sleekly-designed music player and easy-to-use music management program. Initially, it was able to sign five major music companies, starting with Warner Music. They agreed Apple could sell single songs for 99 cents each, shared 30% and 70% between Apple and the music companies, respectively. While Apple insisted that it made little profit from iTunes itself, it captured value with it by providing massive content for its complementary, high-margin hardware business (first the iPod and then the iPhone).

Why did the music industry let Apple capture so much value? Apple attracted the music industry with an elegant and functional beta version of its software. Moreover, Apple hardware was used widely in the music industry. Steve Jobs, CEO of Apple at that time, reported that record companies lacked technological knowhow and did not know what to make of changes that came along with the internet and Napster (Godell (2011)). With some concessions on digital rights management that turned out to be a blessing for Apple, as it meant that the music could only be played on Apple devices, and the music industry's belief that it would be Mac-only (it was suddenly opened for the Microsoft world half a year after the launch), Apple was able to persuade the music industry to license their content through the iTunes Store (Hill (2013)).

The case points to a tight relation between Apple's ability to capture value and the music industry's mistaken beliefs about how successful iTunes would become. We can analyze this value capture with the tools introduced in this section. In our stylized model, we have two agents, Apple A and Music Company M, and a simple state space with two states, hit, h, and status quo, q. There are type mappings for each agent formalizing their beliefs. Finally, there is a state-contingent characteristic function modeling the value created by the agents alone and together in both states.

[24] We thank the senior editor, Timo Ehrig, for already suggesting this example in 2012.

Information is such that Apple believes in its success no matter what. That is, its type mapping is given by $t_A(h)(\{h\}) = t_A(q)(\{h\}) = 1$. In both states, it is certain of its success. The type mapping of Music Company is given by $t_M(h)(\{q\}) = t_M(q)(\{q\}) = p$. That is, in both states, Music Company believes in the status quo with probability p. The information structure is depicted in Figure 5.

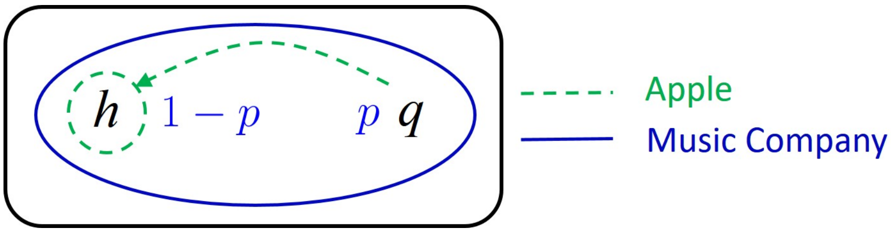

Figure 5: Information Structure of Apple iTunes vs. Music Company

Music Company's type mapping is given by the blue oval, indicating the support of its belief at both states with probabilities written beside the states, respectively. Apple's type mapping is given by the green dashed arrow and circle. It indicates that at both states, Apple is certain of h. For instance, the green dashed arrow emanating from state q leads to the circle around h signifying the fact that at q Apple is certain of h.

Finally, let us make assumptions about the characteristic function. Each agent knows that in any state of the world, Apple can generate a value of 10 on its own and the Music Industry can generate a value of 30 on its own. When a partnership between the two is formed, it may either be successful (h) or may just continue the status quo (q). A success generates a total value created of 100, and the status quo generates 40. The characteristic function is given by Table 2.

Groups	h	q
{Apple, Music Company}	100	40
{Apple}	10	10
{Music Company}	30	30

Table 2: State-Dependent Characteristic Function for Apple iTunes versus Music Company

What distributions of value are in the coarse core? For now, assume that Music Company believes in success or status quo with equal probability (i.e., fix $p = \frac{1}{2}$, which seems optimistic given the background information sketched above). With that, let us begin by checking the group blocking condition. For a distribution of value π to be in the coarse core, there can be no groups that meet the blocking requirements in any state. To check this, we must look at the expected value of payoffs according to the beliefs of the agents in that state (given by their types) and ensure that there is no alternative distribution that would be feasible for the group and result in strictly greater payoffs for all. The expected payoffs from distribution $\pi = (\pi_A, \pi_M)$ are for Apple and the Music Company, respectively,

$$\mathbb{E}[\pi_A \mid t_A(h)] = \mathbb{E}[\pi_A \mid t_A(q)] = \pi_A(h)$$

$$\mathbb{E}[\pi_M \mid t_M(h)] = \mathbb{E}[\pi_M \mid t_M(q)] = \frac{1}{2}\pi_M(h) + \frac{1}{2}\pi_M(q)$$

Begin with the group {Apple, Music Company}. Apple's expected value increases when dollars are shifted to state h from state q. Music Company's expectations are such that it is indifferent between dollars received in either state. Therefore, as a starting point, suppose π distributes all the value to Apple in h and all the value to Music Company in q. Under this distribution,

$$\mathbb{E}[\pi_A \mid t_A(h)] = \mathbb{E}[\pi_A \mid t_A(q)] = 100$$
$$\mathbb{E}[\pi_M \mid t_M(h)] = \mathbb{E}[\pi_M \mid t_M(q)] = 20$$

Notice that this distribution maximizes the aggregate expected payoff in both states: transferring \$1 from A to M in h reduces A's expected payoff by \$1 while increasing M's expected payoff by only \$0.50. Alternatively, transferring \$1 from A to M in q has no effect on A's expected payoff but increases M's by \$0.50. This distribution creates the greatest aggregate expected value. There is no way to shift value around to make both agents strictly better off.

So far so good. For the coarse core, however, we also need to check that none of the agents individually block the proposal. Apple must get an expected payoff of at least 10 because this is the amount it can get for certain on its own. Similarly, Music Company knows it can capture 30 on its own. Therefore, the preceding distribution is blocked by Music Company (since that distribution yields it an expected payoff of only 20). To eliminate this blocking group, 20 must also be allocated to Music Company in state h, thereby yielding it an expected value of $\frac{1}{2}20 + \frac{1}{2}40 = 30$. The following distribution passes the no blocking groups condition: Apple receives 80 in h for an expected payoff of 80; and Music Company receives 20 in h and 40 in q for an expected payoff of 30. Neither agent can do better individually and there is no way to redistribute value so as to make both agents strictly better off. Indeed, any allocation in the coarse core must give all the value at q to Music Company. Any other arrangement that yields an expected payoff of at least 30 to Music Company – say, giving Apple 40 in q and Music Company 60 in h can be rearranged to make both agents strictly better off by trading value from Music Company to Apple in h in return for value to Music Company in q.

Finally, we need to check feasibility. Value capture in the coarse core cannot be more than 100 at h and more than 40 at q. Taking all these conditions together, π is in the coarse core if:

$$\pi_M(q) = 40, \pi_M(h) \geq 20, \pi_A(q) = 0, \pi_A(h) = 100 - \pi_M(h).$$

Thus, the distribution of 80 for Apple in h, 0 for the Apple in q, 20 for Music Company in h, and 40 for Music Company in q is in the coarse core.

Interestingly, this gives Apple an expected profit of 80, which is greater than Apple's expected added value of 70. To understand why, note that Apple's added value at h is $100 - 30 = 70$. Since Apple is certain of h, this is also its expected added value. However, Apple can also earn 10 by agreeing to let Music Company keep more profit if the status quo obtains. Apple does not think the status quo will obtain, hence their asymmetric information allows them to trade off lower profits in that state versus higher profits in the state h that Apple is certain will occur.[25] Obtaining a profit that exceeds one's added value is in contrast to value capture

[25] Why did Cirque earn no positive profits in the incomplete information version of the opening example? With no expected added value, the no losses condition prevented Cirque from earning expected payoffs by "betting" on states it found uniquely likely to occur.

under complete information: it is well known that under complete information the added value of an agent serves as an upper bound on the value captured by that agent (Brandenburger and Stuart, 1996, p. 13). We will analyze this further in Section 4.3.

Let us now conduct some comparative statics analysis of Apple's value capture with respect to Music Company's belief, p, in the status quo. In the coarse core, the minimum Apple is assured is the value it can capture independently, 10. The maximum is determined by the expected capture of Music Company. As long as the expected capture from π is above 30, Music Company will not block the agreement. This yields the inequality,

$$(1-p)\pi_M(h) + p\pi_M(q) \geq 30.$$

We also know that $\pi_M(q) = 40$ as long as $p > 0$. That is, Apple leaves the entire value to Music Company in the status quo state simply because it is certain of a hit, state h. Finally, the coarse core requires that all value created be captured in both states. This implies,

$$\pi_M(h) = 100 - \pi_A(h).$$

Putting all conditions together yields an equation for Apple's maximum value capture at h in the coarse core as a function of Music Company's belief $p \in (0, 1]$ in the status quo:

$$\max \pi_A(h) = 100 - 40\frac{p}{1-p} - 30\frac{1}{1-p}.$$

This is also Apple's expected maximum value capture in the coarse core since Apple believes h is sure to occur. The range of the minimum and maximum profit in the core has been labeled as the *core interval* in the literature (MacDonald and Ryall, 2004). Analogously, we can consider Apple's coarse core interval. Figure 6 plots Apple's coarse core interval as a function of Music Company's belief p in the status quo.

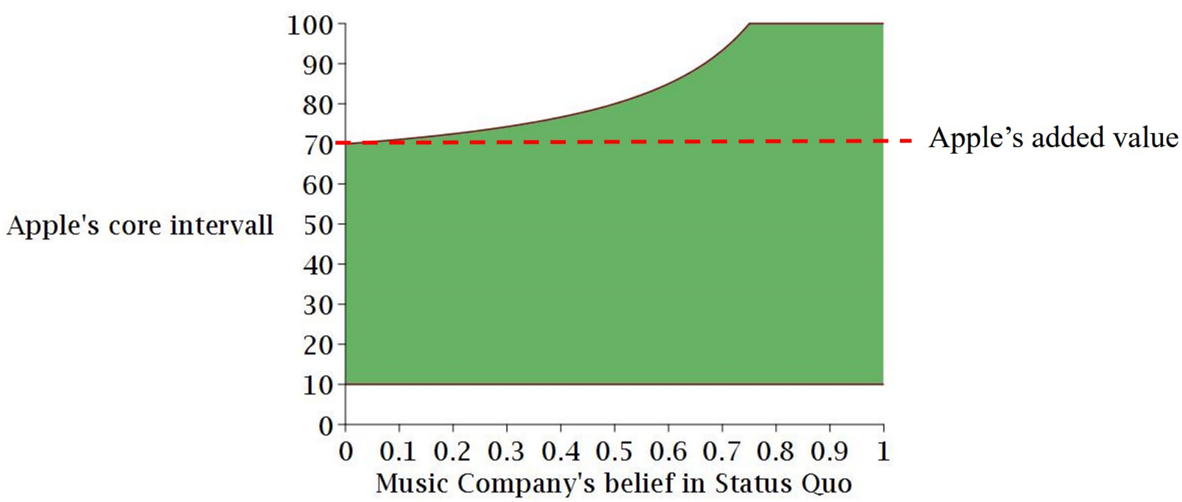

Figure 6: Apple's Value-Capture Possibilities as a Function of Music Company's Belief in the Status Quo

We observe that over the entire range, Apple could capture more value than its own expected added value. Moreover, Apple's maximum value capture in the coarse core is increasing in Music

Company's belief on the status quo. Thus, Apple had no interest in changing beliefs in the music industry. In fact, belief inertia in the music industry helped Apple capture a large share of the value. This matches nicely with some anecdotal evidence from the negotiations between Steve Jobs and the music industry: Jobs reportedly complained 'how the music business just didn't get it' (Knopper (2011)). While Apple's capabilities of designing beautiful hardware and functional software certainly helped convince the music industry to deal, its extraordinary profits did appear to depend on the skeptical beliefs of its "coopetitors" in the music industry. This highlights how internal resources and capabilities interact with external elements that rest with competitors to give rise to competitive advantages. This case illustrates Barney (1986)'s conjecture that asymmetric information and, consequently, heterogeneous expectations may allow some firms to reap supernormal profits from strategic resources. Indeed, our tools go beyond this to show, quantitatively, how such supernormal profits depend upon asymmetric information (in a very simple example).

4 Value Capture under Unknown Unknowns

Cooperative games of incomplete information allow us to model asymmetric uncertainty of agents with respect to events that are pertinent to their value creation opportunities. Yet, as illustrated by our introductory example, business strategies based upon superior creativity, foresight, imagination, ideas, innovation, and alertness can lead to positive economic profits. These strategies exploit the unknown unknowns of one's rivals, aspects of the world of which those rivals are simply unaware. This seems to be the type of situation Penrose (1959, Chapter III) had in mind when she emphasized the importance of the "*subjective* productive opportunity of the firm" for explaining systematic performance heterogeneity. In this section, we introduce general tools for rigorous analysis of unawareness in market settings.

4.1 Unawareness Structures

In general, the modeling of unknown unknowns has proven to be a challenge. Importantly, Dekel et al. (1998) demonstrate that any decision theory featuring a standard state space (e.g., subjective expected utility theory and non-expected utility theory, like maximin expected utility or Choquet expected utility used for ambiguity analyses) is inadequate to the task.[26] To model unawareness one must go beyond a state space. There are several approaches to doing so, developed in artificial intelligence, logic, and game theory (see Schipper (2015) for a survey).

[26] To understand this impossibility result at an intuitive level, lets consider what states really mean. We use states to let 'something' depend on them (e.g., like the value created by firms). So whatever effects this 'something' is implicitly described by states even though states are often just taken as some abstract set of indices or points in some space. Taking this thought more seriously, we can think of a state as a comprehensive description of all relevant events like "drug A cures X, X is caused by Y, ...". Now given that a decision maker can reason like in this *one* state, she could also use her language to reason about alternative states like the state "drug A cures X, X is *not* caused by Y, ...". More generally, the language to express the events that occur in one state, can be used to reason about occurrence and non-occurrence of any events in the language, and thus describe any state referring to those events. Consequently, a decision maker whose reasoning is modeled implicitly with a state space can not be unaware of any event expressible in this state space because implicitly she has the language to reason about all events modeled in this state space.

Some of these approaches require the user to know epistemic logic, which, unfortunately, is not in the repertoire of most social scientists.

Fortunately, there is one approach that does not require the user to know epistemic logic (even though it can be cast in those terms) – the unawareness structures developed by Heifetz et al. (2006), Heifetz et al. (2008), and Heifetz et al. (2013a). This is the approach we adapt to cooperative games. It is illustrated in our earlier Cirque du Soleil example. The solution is to use *more than one* state space. Intuitively, by allowing more than one state space we can allow for differently rich perceptions of the world's possibilities. When an agent forms beliefs over events in one space (like B and P in the Red Ocean Space), she is blind to some events expressed in the richer space (like the Blue Ocean Space). This simple idea is generalized by unawareness structures.

Begin with a finite *set of state spaces* $\mathcal{S} = \{S, S', S'', ...\}$. We assume that each state space is nonempty and finite. Moreover, we assume that any two state spaces are disjoint. Importantly, they are ordered according to the relative richness of their descriptions of the world's possibilities. This order is not necessarily complete because one event may be described in space S but not in space S' while a different event may be described in space S' but not in S. However, for any two spaces we can consider a (weakly) richer space that describes all events that can be described in either space and a (weakly) poorer space that describes all events that can be described in both spaces. In mathematical terms, we require a lattice of spaces $\langle \mathcal{S}, \succeq \rangle$, where $\succeq$ is a partial order on the spaces in $\mathcal{S}$. It is also useful to have notation for all states, no matter in which space: let $\Omega = S \cup S' \cup S'' \cup ...$ denote the *union of spaces* in $\mathcal{S}$.

State spaces are interrelated. In our Cirque example, the states "novel synthesis a hit" and "novel synthesis a flop" in the Blue Ocean Space are refinements of the state "red ocean" in the Red Ocean Space – the possibilities pertaining to the novel synthesis are not discernible in the simpler space, where the novel synthesis is not even imagined. To model those relations, we introduce projections from more expressive to less expressive spaces. Formally, for any two spaces $S, S' \in \mathcal{S}$ with $S' \succeq S$, we define a function $r_S^{S'} : S' \longrightarrow S$ that maps states in the more expressive space S' to states in the less expressive space S. Interpret projections as "erasing" some details from the description of a state in the richer space to yield a somewhat impoverished state description in the poorer space.

Projections are assumed to be onto: every state in the poorer space has a state in the richer space that is related to it. In the general setting there can be many spaces that are ordered, for instance $S'' \succeq S' \succeq S$. This raises the need to impose some consistency on projections. Specifically, we require that the projection from S'' to S yield the same as the projection from S'' to S' followed by the projection from S' to S: $r_S^{S''} = r_S^{S'} \circ r_{S'}^{S''}$.

Projections map states in richer state spaces to those in poorer state spaces. Based upon these, it is also possible to identify all the states in richer spaces that are related to a state in a poorer space. For any two spaces $S, S' \in \mathcal{S}$ with $S' \succeq S$ and state $\omega \in S$, let $\rho_S^{S'}(\omega)$ denote the subset of states in S' that project to ω: $\rho_S^{S'}(\omega) \equiv (r_S^{S'})^{-1}(\omega)$, the inverse image of ω under $r_S^{S'}$. We call $\rho_S^{S'}(\omega)$ the *ramifications of* ω *in* S'.

At this point, it may be helpful to illustrate the formalism introduced so far. Consider Figure 7. There are four spaces. In the upmost space, S_{pq}, there are states that describe whether or not event p or event q happens. For instance, at state pq both event p and event q

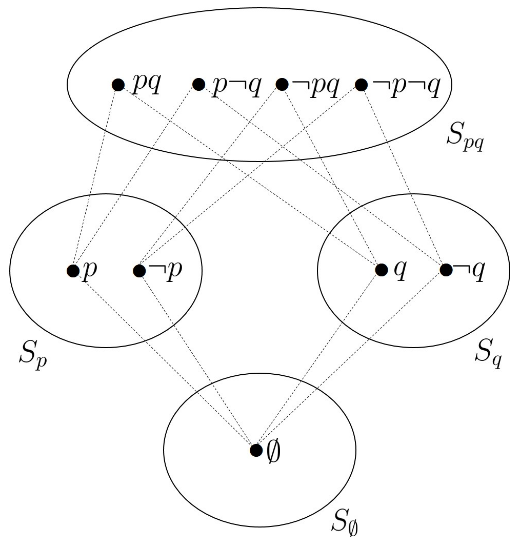

Figure 7: Lattice of Spaces and Projections

happen. At state $p\neg q$ event p happens but q does not happen, and so on.[27] The leftmost space S_p is poorer than space S_{pq} because nothing can be expressed with regard to event q. Similarly, the rightmost space S_q is poorer than S_{pq} because nothing can be expressed with regard to event p. Spaces S_p and S_q are incomparable to each other with respect to their expressiveness: S_p can express whether or not p happens but S_q cannot; and S_q can express whether or not q happens but S_p cannot. Finally, there is a least expressive space, $S_\emptyset$, in which none of these details can be elaborated. This is a simple example of a nontrivial lattice of spaces.[28] We also indicate in Figure 7 the projections from higher to lower spaces by dashed lines. For instance, state pq in space S_{pq} projects to state p in space S_p but also to state q in space S_q. In this example, the relationships between states in different spaces are obvious due to the labels we have chosen.

In a standard state space, an event corresponds to subset of states. In the current setting with multiple state spaces, if there is a space in which an event obtains in some states, then also at elaborations of those states in more expressive spaces this event obtains (and may be others as well). This follows from the lattice order of spaces and the projections. For instance, in Figure 7, let p stand for "penicillin has antibiotic properties". Then also at pq we have p, "penicillin has antibiotic properties" and some q like "penicillin can be synthesized for mass production". Formally, take any subset of states $D \subseteq S$ in some space $S \in \mathcal{S}$. Denote by $D^{\uparrow} \equiv \bigcup_{S' \succeq S} \rho_S^{S'}(D)$ the union of elaborations of D in more expressive spaces. This is the union of elaborations of D over all spaces (weakly) more expressive than S (i.e., all states in spaces where what is implicitly described in $D \subseteq S$ can also be described). An *event* E in the lattice structure has the form $E = D^{\uparrow}$ for some set of states D in some space S. We call D the *base* of the event E and S the *base-space* of the event. The base space of an event E is denoted by $S(E)$.

We illustrate the notion of event in Figure 8. It depicts the same lattice of spaces as in

[27]The"¬" symbol stands for "not".

[28]For illustration purposes, we simply index spaces by primitive events that can be expressed in them.

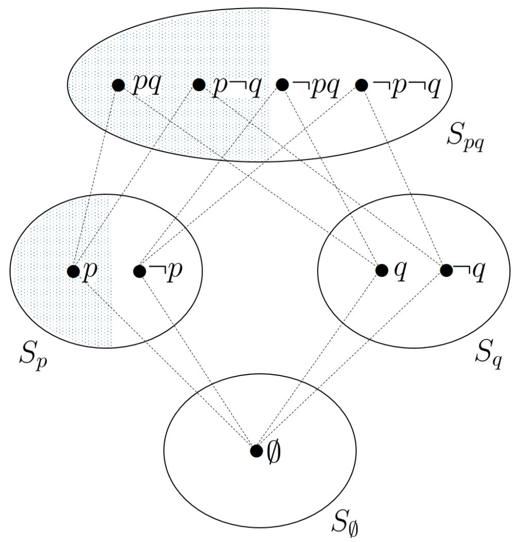

Figure 8: Event $[p]$

Figure 7. Consider now all states in space S_p in which p obtains. This is only the state p in space S_p. We indicate it with the dotted area in space S_p. Now consider the elaboration of this set in the more expressive space S_{pq} using the projections. This collects states pq and $p\neg q$, the dotted area in space S_{pq}. The union of these two dotted areas represents the event p (we may write it as $[p]$ in order to distinguish it from the state or the description). It comprises of all the states in all spaces in which p obtains. For this event, $\{p\} \subset S_p$ is the base and S_p is the base space.

Note well that, according to the preceding definition, not every subset of states is an event. For instance, the (singleton) set of states $\{p\}$ is not an event because it misses the elaborations in more expressive spaces. Alternatively, the union of $\{p\}$ and $\{q\}$ is not an event because there is no unique base space that contains p and q. Let Σ denote the *set of events*.

Like in the case of incomplete information, we proceed by introducing probability distributions on state-spaces. For any state space $S \in \mathcal{S}$, let $\Delta(S)$ be the set of probability distributions on S. Even though we consider probability distributions on each space $S \in \mathcal{S}$, we can talk about probability of events that, as we just have seen, are defined across spaces. To extend probabilities to events of our lattice structure, let S_μ denote the space on which μ is a probability measure. Whenever for some event $E \in \Sigma$ we have $S_\mu \succeq S(E)$ (i.e., the event E can be expressed in space S_μ) then we abuse notation slightly and write

$$\mu(E) = \mu(E \cap S_\mu).$$

If $S(E) \not\preceq S_\mu$ (i.e, the event E is not expressible in the space S_μ because either S_μ is strictly poorer than $S(E)$ or S_μ and $S(E)$ are incomparable), then we leave $\mu(E)$ undefined.

To model an agent's awareness of events and beliefs over events and awareness and beliefs of other groups, we introduce type mappings. Given the preceding paragraph, we see how the belief of an agent at state $\omega \in S$ may be described by a probability distribution over states in a less expressive space S' (i.e., $S \succeq S'$). This would represent an agent who is unaware of the events that can be expressed in S but not in S'. These events are "out of mind" for him in the sense that he does not even form beliefs about them at ω: his beliefs are restricted to a space that cannot express these events.

More formally, for every agent $i \in N$ there is a *type mapping* $t_i : \Omega \longrightarrow \bigcup_{S \in \mathcal{S}} \Delta(S)$. That is, the type mapping of agent $i \in N$ assigns to each state $\omega \in \Omega$ of the lattice a probability distribution over some space. Now a state does not only specify which events affecting value creation may obtain, and which beliefs agents hold over those events, but also which events agents are aware of. Recall that S_μ is the space on which μ is a probability distribution. Since $t_i(\omega)$ now refers to agent i's probabilistic belief in state ω, we can write $S_{t_i(\omega)}$ as the space on which $t_i(\omega)$ is a probability distribution. $S_{t_i(\omega)}$ represents the *awareness level* of agent i at state ω. This terminology is intuitive because at ω agent i forms beliefs about *all* events in $S_{t_i(\omega)}$.

For a type mapping to make sense, certain properties must be satisfied. The most immediate one is *Confinement:* if $\omega \in S'$ then $t_i(\omega) \in \triangle(S)$ for some $S \preceq S'$. That is, the space over which agent i has beliefs in ω is weakly less expressive than the space contains that ω. Obviously, a state in a less expressive space cannot describe beliefs over events that can only be expressed in a richer space. We also impose Introspection, which played a role in our prior discussion of incomplete information: every agent at every state is certain of her beliefs at that state. In Appendix A, we discuss additional properties that guarantee the consistent fit of beliefs and awareness across different state-spaces and rule out mistakes in information processing.

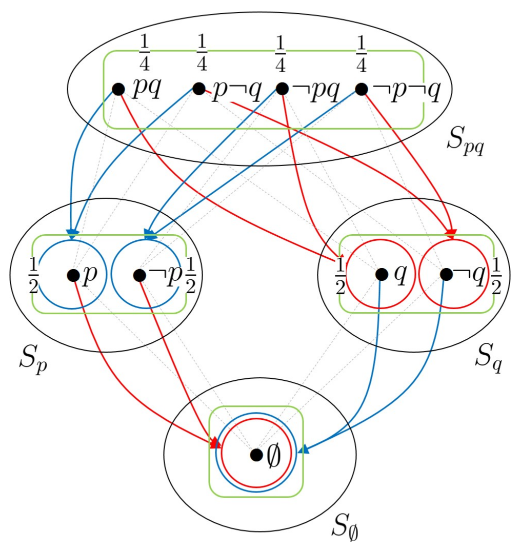

Figure 9: Unawareness structure

It might be helpful to illustrate type mappings with an example. Figure 9 depicts the same lattice of spaces as in Figures 7 and 8. In addition, we depict the type mappings for three different groups. At any state in the upmost space S_{pq}, the blue agent is aware of p but unaware of q. Moreover, she is certain whether or not p depending on whether or not p obtains. This is modeled by her type mapping that assigns probability 1 to state p in every state where p obtains and probability 1 to state $\neg p$ in every state where $\neg p$ obtains. (The blue circles represent the support of her probability distribution that must assign probability 1 to the unique state in the support.) An analogous interpretation applies to the red agent except that she is an expert in q.

In contrast, the green agent is aware of both p and q but knows nothing with certainty, modeled by her probabilistic beliefs in the upmost space that assigns equal probability to each state in it.[29]

Unawareness structures allow us to model an agent's awareness and beliefs about another agent's awareness and beliefs, beliefs about that, and so on. This is because, as in the incomplete information case, beliefs are over states and states also describe the awareness and beliefs of groups. Return to Figure 9. At state pq the green agent assigns probability 1 that the blue group is aware of p but unaware of q. Moreover, he assigns probability 1 to the blue agent believing with probability 1 that the red group is unaware of p.[30]

4.2 Value Creation under Incomplete Information and Unawareness

The preceding structure provides a foundation by which to model the reasoning of agents about known and unknown unknowns. To model *value creation and capture* under known and unknown unknowns, we must augment the formalism with cooperative game theory. The good news is that the extension of cooperative games with incomplete information to those with unawareness is now surprisingly straightforward. We simply replace the state space and type mappings in the definition of cooperative game with incomplete information (Definition 1 in Section 3) by an unawareness structure. The definitions follow almost verbatim!

As before, the characteristic function is state-dependent. However, instead of depending on a state in a single state space, the characteristic function can depend on any state in any state space in the lattice of spaces. Formally, $v : \Omega \times \mathcal{G} \longrightarrow \mathbb{R}_+$ assigns to each state $\omega \in \Omega$ (i.e., in the union of spaces) and each group $G \in \mathcal{G}$ a nonnegative value $v(\omega, G) \geq 0$. As before, $v(\omega, G)$ is the "ex-post value created by G if the events described in ω obtain." The new wrinkle is that $v(\omega, G)$ is the actual value created ex-post *as perceived in the mind of someone whose awareness level is given by the state space that contains* ω. Because ω may now leave out important features of the world that affect the amount of value created, this perception may not be fully accurate.

One may be tempted to impose conditions on how values are related across spaces. For instance, in the Cirque du Soleil example, there was a state in the Blue Ocean Space, f, that projected to the state r in the Red Ocean Space. Moreover, the values for various agents at f coincided with the values of various agents at r. Alternatively, in some situations the value at a state in a poorer space may be the average (with respect to some prior) of the values in some more expressive space (i.e., agents miss some events, but are correct on average). These alternative assumptions on how values are related across spaces may differ with the applications and do not seem to be perfectly general. Hence, we refrain from imposing further assumptions on the characteristic function in our general set-up and leave it to the researcher to fill in the appropriate characteristic function values consistent with the application being modeled.

[29] The example is taken from Schipper (2016) who shows how a generalist (i.e., the green agent) emerges as an entrepreneur and forms a firm made of specialists (i.e., the blue or red agents) in a knowledge-belief and awareness-based theory of the firm using strategic network formations games under incomplete information and unawareness.

[30] We note, it has been shown that under appropriate assumptions on spaces $S \in \mathcal{S}$ and the type mapping, unawareness structures are rich enough to model any higher order beliefs of agents (see the working paper version of Heifetz et al. (2013a)).

At the interim stage, when agents form their beliefs given their awareness, they can consider expected value creation. Of course, the expectations about the potentials of groups to create value may differ across agents, depending on their awareness and beliefs. Moreover, since unawareness structures allow for reasoning by agents about other agents' beliefs and awareness, an agent may realize that another agent expects different value creation because she knows that this agent is unaware of some events that affect it. This is the case for C in the Cirque du Soleil example, who is certain that B and P are both unaware of the Blue Ocean and, consequently, do not perceive the opportunity of the novel form of entertainment.

We summarize our discussion in the definition of cooperative game with incomplete information and unawareness:

Definition 5 *A finite cooperative game with incomplete information and unawareness* $\left\langle N, v, \langle \mathcal{S}, \succeq \rangle, \left(r^S_{S'}\right)_{S' \preceq S}, t_1, ..., t_n \right\rangle$ *consists of*

- *a finite set of agents* N,
- *a finite unawareness structure* $\left\langle \mathcal{S}, \left(r^S_{S'}\right)_{S' \preceq S}, t_1, ..., t_n \right\rangle$, *where* $\mathcal{S}$ *lattice of finite state spaces with projections* $r^{S'}_S : S' \longrightarrow S$ *for any* $S' \succeq S$, $S, S' \in \mathcal{S}$, $\Omega = \bigcup_{S \in \mathcal{S}} S$, *and for each agent* $i \in N$, $t_i : \Omega \longrightarrow \bigcup_{S \in \mathcal{S}} \Delta(S)$ *is a type mapping satisfying properties outlined in Appendix A.*
- *a state-contingent characteristic function* $v : \Omega \times \mathcal{G} \longrightarrow \mathbb{R}_+$ *with* $\mathcal{G} = \{G \subseteq N \mid G \neq \emptyset\}$.

It should be clear that cooperative games with incomplete information are a special case of cooperative games with incomplete information and unawareness in which the lattice structure contains just a single state space.

4.3 Value Capture under Incomplete Information and Unawareness

Given cooperative games with incomplete information and unawareness, the theory of value capture under incomplete information and unawareness is now also a straightforward extension of value capture under incomplete information. Distributions of value are now defined for each state in the entire lattice structure. That is, $\pi : \Omega \longrightarrow \mathbb{R}^N_+$ is a function that assigns to each state $\omega \in \Omega$ a distribution value of $\pi(\omega)$. As before, $\pi(\omega) = (\pi_1(\omega), ...\pi_n(\omega))$ is a vector of state-contingent quantities of value captured, one for each agent $i \in N$. Since ω may be just a partial description of events that actually happen (because it is in a poorer space), $\pi_i(\omega)$ may not be the precise amount of value captured by agent i but, instead, the capture of agent i as perceived by someone whose awareness level corresponds to the space that contains ω.

In some applications, it is makes sense to impose the following condition on distributions: Distribution π is *private-awareness measurable* if for all $i \in N$ and $\omega \in \Omega$ we have $\pi_i(\omega) = \pi_i(\omega_{S_{t_i(\omega)}})$. This condition ensures that the payoff i receives at a state ω is the same as the one she anticipates receiving at the corresponding state given her awareness at ω. To see the idea, recall that $S_{t_i(\omega)}$ represents the awareness level of agent i at state ω. If ω occurs, she can reason about all events in $S_{t_i(\omega)}$ and thus is aware of $\omega_{S_{t_i(\omega)}}$, the state that corresponds to ω but misses out all events that she is unaware at awareness level $S_{t_i(\omega)}$. The condition ensures now that her payoff at ω must be equal to her payoff at $\omega_{S_{t_i(\omega)}}$. The idea is that at every state, agent i

consents only to payoff consequences emanating from events she is aware of at that state. She can not be forced to bear consequences from events she has been unaware of at the contracting time.

This condition may or may not be justified in applications. It is likely justified when an agent can walk away from a deal any time after the contracting stage. For instance, in the Cirque du Soleil example, we imposed this condition taking the view that B and P could resign their positions at C's venture any time after the contracting stage and resort to what they have been doing previously. This condition is less justified in contracting situations where one party committed to a deal without perfect indemnifications for all events she is unaware of. There is a distinction between free form *contracting* and free form splits of value across states. Contracting implies that agents can ex-ante describe the conditions under which the contract, whatever it may be, will pay out, and the conditions under which the contract is incomplete. Splits of value imply that even in situations where some agents cannot foresee an event, value is nonetheless created, and the modeler assigns through the characteristic function a value that depends implicitly on what actions agents would take given their awareness. Our model applies to both situations, depending on whether the measurability assumption is retained. Our general theory and results do not rely on it and apply with and without this condition.[31]

To identify reasonable value distributions, we extend the solution concept of the coarse core to this setting. The definitions developed in Section 3 for cooperative games with incomplete information apply verbatim to cooperative games with incomplete information and unawareness once we replace the state space with the union of spaces in the unawareness structure and use the type mappings as defined for the unawareness structure. We the sake of completeness we (re)state it here again:

Definition 6 (Coarse Core) *The profile of payoff functions π is in the coarse core of the cooperative game with incomplete information and unawareness $\left\langle N, v, \langle \mathcal{S}, \succeq \rangle, \left(r^{S}_{S'}\right)_{S' \preceq S}, t_1, ..., t_n \right\rangle$ if and only if*

(i) π is feasible for the entire set of agents N, and

(ii) for all $\omega \in \Omega$ no group blocks π.

This extension brings about additional conceptual features. In particular, when agents contemplate whether a group blocks a distribution of value, not only is no information revealed among agents, but neither is any awareness. We refer the interested reader to Appendix A in which write out the formal detail of our approach. For an example illustrating our approach, we refer to our analysis of Cirque due Soleil in Section 2. There we realized already that P and B would block any distribution of payoffs in which they do not earn 40 as a group because at any state it is common certainty among them that they can create at least 40 as a group.

We demonstrate the applicability of our framework with four general observations on value-capture and added value. Brandenburger and Stuart Jr. (1996, p. 13) observe that an agent's added value places an upper bound on the amount of value that the agent can capture in the core under complete information and common awareness. We have seen in the Cirque du Soleil

[31] Note that the Cirque example implicitly relied on this measurability assumption in constructing core payoffs.

case that under unawareness an agent may have zero added value but capture a strictly positive amount of value in the coarse core. Similarly, in the Apple iTunes case we have seen that an agent may capture a higher expected value than her expected added value. Thus we conclude:

Proposition 1 *It is possible under incomplete information or unawareness that an agent captures an expected value higher than her expected added value.*

However, observe in the Apple iTunes case that Apple thinks that it captures value strictly above its expected added value, while Music Company thinks that Apple earns its expected added value. There is no agreement in the sense of common certainty that Apple captures value above its added value. This suggests that while the observation in Brandenburger and Stuart Jr. (1996) does not extend to value capture under incomplete information or unawareness, we can nevertheless prove the following generalization of Brandenburger and Stuart's observation:

Proposition 2 *For any agent $i \in N$, in the coarse core it cannot be common certainty that everybody expects i to obtain profits strictly above her added value.*

The formal statement and proof are relegated to the Appendix C. The observation highlights the role of common certainty of expectations. Clearly, this yields Brandenburger and Stuart's observation in the special case of complete information since common certainty is trivially satisfied under complete information.

MacDonald and Ryall (2004, Proposition 1) observe that in order for an agent to capture strictly positive value in the core under complete information and common awareness, it is necessary that she have a strictly positive added value. Again, we have seen in the Cirque du Soleil case that an agent can have zero added value and still capture strictly positive value in the coarse core. Thus, we conclude:

Proposition 3 *It is possible under incomplete information or unawareness that an agent captures strictly positive expected value while having an expected added value of zero.*

While McDonald and Ryall's observation does not extend to value capture under incomplete information and unawareness, we can nevertheless prove a generalization of McDonald and Ryall's observation:

Proposition 4 *For any agent $i \in N$, in the coarse core it cannot be common certainty that everybody expects i to have zero added value and to obtain strict positive profits.*

Again, the formal statement and proof is relegated to Appendix C. Proposition 4 can be illustrated with the Cirque du Soleil case. While Cirque adds zero value and expects to capture strict positive value, this is not common certainty among all agents. In the unawareness structure of the example in Figure 1, only the universal event comprising of all states is common certainty among all agents. But it is not the case that at all states of that event Cirque captures strict positive value. E.g, at state r all agents believe that Cirque captures zero value. In fact, this is common certainty at r. Both at h and f, agents B and P believe that Cirque captures zero value while only Cirque believes in capturing strict positive expected value. Hence, there is no state at which it is common certainty that Cirque captures strict positive expected value.

4.4 Unawareness of the Presence of Other Agents

The framework we outlined so far allows for incomplete information and unawareness of events relevant for value creation and value capture, but we did not explicitly consider unawareness of other agents who are, indeed, present in the market. Menon (2018) argues compellingly for the need to consider unawareness of the presence of other agents in value capture theory. An example in which a firm was unaware of the presence of a competitor is British Satellite Broadcasting's unawareness of Sky TV when rolling out their satellite TV in the UK (Menon (2018); see Ghemawat (1993) for a case study).

Our general framework in Appendix A features unawareness of the presence of other agents as well as of relevant events. This has interesting effects because groups blocking certain distributions of value can only be formed among agents who are aware of each other.[32] Menon (2018) discusses an example of unawareness in which one agent is unaware of the other agents who, themselves, are aware of everyone. This sort of one-sided unawareness is easy to model in our framework (using only two state spaces). Here we illustrate our approach with an example involving double-sided unawareness of agents.

To set the stage, consider one supplier, S1, having cost 50, who can supply at most to one firm. There are two firms, F1 and F2, each having cost 10. Finally, there is one buyer, B, who buys from at most one firm. The willingness to pay of the buyer is 100 no matter whether it buys from F1 or F2. The value created by any subgroup made up of S1, B, and either firm is therefore 40. Since both firms have added value of zero, with complete information they never capture positive value in any core allocation. For example, one distribution in the core is 20 for B, 0 for both F1 and F2, and 20 for S1. This distribution could be implemented with a price of 80 paid by B to F1, a price of 70 paid by F1 to S1, and F1 delivering the good to B. Note that with this distribution F2 stays out of the market, yet its competitive threat is what ensures that F1 earns no profit. Let us call this the status quo situation.

Now consider the situation in which F2, perhaps a foreign firm, is aware of an alternative foreign supplier, S2, whose cost is just 20, while domestic agents S1 and F1 are both unaware of S2. We assume that B is also aware of S2 and F2. S2 is aware of F2 but unaware of all other agents. This can be represented by an unawareness structure with four spaces, S_0, S_1, S_2, and S_3, with $S_3 \succeq S_1 \succeq S_0$, $S_3 \succeq S_2 \succeq S_0$, and S_1 and S_2 being incomparable. Each space is a singleton, as shown in Figure 10, hence projections are trivial in this example. In the state of S_1, agents S1, F1, F2, and B exist. In the state of S_2, agents S2 and F2 exist. In the state of S_3 all agents exist. Finally, agent F2 exists in all spaces and states.[33] Thus, the event in which

[32] The coarse core of a cooperative game with unawareness potentially addresses a concern that has been raised with the use of the core as a solution concept in value capture theory. Montez et al. (2018) point out that "(i)n practice, however, the core suffers from its own drawbacks. The most significant of these is that its complexity increases exponentially with the size of N." Further they write that "the agents in N are not only required to grasp an enormous number of different, potential ways of organizing their economic activities, but also to know both the values that these would produce and, simultaneously, the shares of V captured by all of the other market participants for the activities actually undertaken. In most settings, this seems far beyond the ken of any real human being." If agents in real life with a bounded perception are unaware of some agents, then naturally they do not consider all groups of market participants. This is precisely the case in the coarse core under unawareness of the presence of some agents.

[33] In this example, space S_0 is just there to make the structure into lattice and the set of states in which F2 exists into an event.

F2 exists is the universal event. The event in which S1, F1, and B exists is $\{\omega_1, \omega_3\}$ and the event in which S2 exists is $\{\omega_2, \omega_3\}$. The type mapping of F2 is indicated by the dashed red circles. The type mapping of B is given by the orange dot-dashed circles. The type mapping of S1 and F1 is indicated by the solid-lined blue circle and arrow. Finally, the dotted green circle and arrow indicate S2's type mapping. Note that types of agents are only defined at states at which they exist. All type mappings ascribe unit probability beliefs indicated by the fact that each circle contains just one state. Since F2 exists in all states, he is certain of the respective state in each space as shown with the dashed red circles. B is certain of ω_1 and ω_3 in S_1 and S_3, respectively, as shown with the orange dot-dashed circles. For F1 and S1 (solid blue line), at ω_3 their belief is on S_1 signifying the fact that they are unaware of S2 at ω_3. Similarly, at ω_3 the belief of S2 (dotted green line) is on S_2 indicating that he is unaware of B, F1, and S1 at ω_3.

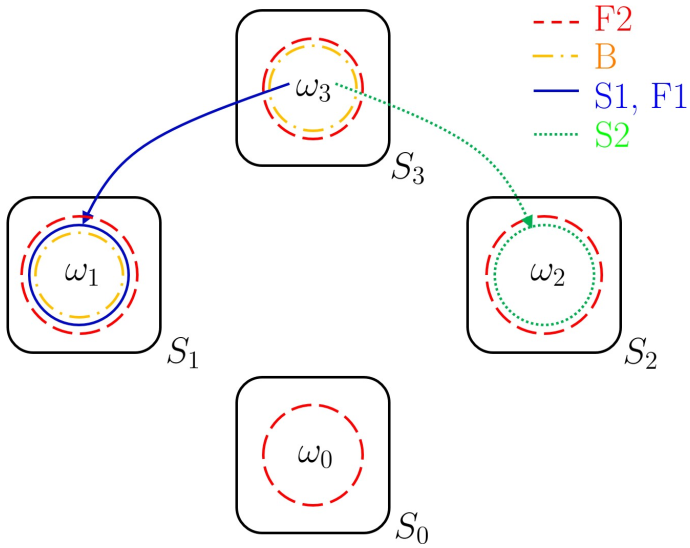

Figure 10: Unawareness Structures in the buyers and sellers example

The characteristic function associates a value with each state and each group of agents that exist at that state. Let costs and willingness-to-pay of agents be the same across states provided that they exist at that state and match our discussion above.

How does asymmetric unawareness of the presence of agents affect distributions in the coarse core? Since F2 is aware of S2 at state ω_3 in S_3, it may now offer to sell to B at a price of 59. B gladly accepts and buys from F2 because F1 and S1 are unable to provide him with a better offer. F2 may purchase inputs from S2 for a price of 20 capturing a value of 29 (the price 59 minus the supply cost of 20 and the firm's own cost of 10). Thus, even though F2 continues to have a zero added value at ω_3, it is possible for F2 to capture a strict positive value in the coarse core due to its superior awareness. *Effectively, F2 enters a seemingly zero-profit market but makes positive profits because it is aware of the more efficient foreign supplier, S2, and that supplier is unaware of F1.*

The distribution of payoffs with $\pi_B(\omega_3) = 41$, $\pi_{F2}(\omega_3) = 29$, $\pi_B(\omega_1) = 40$, and zero for all other agents in all other states in which they exist is in the coarse core. It also satisfies private-awareness measurability: S1 and F1 receive the same payoffs in states of S_1 and S_3, and S2 receives the same payoffs in states of S_2 and S_3. The distribution is feasible at any state. Moreover, at any state there is no blocking coalition among existing agents at that state. It can be implemented at ω_3 with the prices just discussed and at ω_1 with a price of 60 paid by the buyer to F1 and 50 paid by F1 to S1. Clearly, in the actual distribution (namely the one at ω_3), agents S1 and F1 will be surprised that nobody buys from them. Similarly, S2 may wonder why F1 buys from her. They may try to find out what happened and discover the agent(s) of whom they had been previously been unaware. F2 might have anticipated this and procured a long-term commitment from S2 (for instance, with a contractual exclusivity arrangement or upstream integration) to sustain its competitive advantage. We leave dynamic considerations of this kind to future work.

Because the coarse core models *all* distributions of value that are consistent with some form of competition, some of its distributions may not be intuitive at the first glance. For instance, the allocation $\pi_{S1}(\omega_3) = 20$, $\pi_{F1}(\omega_3) = 0$, $\pi_{F2}(\omega_3) = 9$, $\pi_B(\omega_3) = 41$, $\pi_B(\omega_1) = 20$, $\pi_{S1}(\omega_1) = 20$, and 0 for all other agents in all other states is in the coarse core. It is also private-awareness measurable. Here, perhaps counterintuitively, S1 receives 20 in ω_3. However, this is consistent with S1 at ω_1 having received contractual guarantees to capture 20 no matter what. Contracts with upfront fees in addition to payment at delivery are not unusual. One may well ask why the group including B, F2, and S2 cannot block such an agreement at ω_3? The reason is that S2 is not aware of B and, hence, cannot join a blocking group with B and F2 at ω_3. Clearly, in such a situation, B has an incentive to make S2 aware of himself. Yet, if S2 were already committed to a long-term deal with F2 – for instance, through vertical integration – then, S2 is effectively indistinct from F2 when considering subsequent contracting with B. These subtleties are not captured in our detail-free, static approach. As we elaborate in the next section, such considerations are best left to a dynamic approach that extends bi-form games to incomplete information and unawareness.

5 Discussion

In this section, we discuss some important conceptual issues with our framework, its limitations, and opportunities for extending our setting.

5.1 Group Rationality

The coarse core captures the notion of competitive consistency by the absence of blocking groups. This differs from the notion of competitive consistency used in value capture theory under complete information. There, a distribution of values satisfies competitive consistency if, for every group, the total value captured by this group is at least the value created by the group. We refer to this as *group rationality*.[34] Under complete information, it is well-known that both

[34]More formally, consider a characteristic function game with complete information $\langle N, v\rangle$ with a finite set of agents $N = \{1, ..., n\}$ and a characteristic function $v : \mathcal{G} \longrightarrow \mathbb{R}$, whereas previously defined $\mathcal{G}$ denotes the set of nonempty groups of agents. Let $\pi = (\pi_1, ..., \pi_n)$ denote a distribution of value. Since the game has complete information, there is no set of states or state-dependency of the characteristic function or the distribution of values. Competitive consistency of the core means that for any group $G \in \mathcal{G}$, $\sum_{i \in G} \pi_i \geq v(G)$. That is, the sum of values captured by a group is not less than the value produced by the group. This has been called "coalitional

notions of consistency are equivalent. That is, there are equivalent definitions of the core using either the absence of blocking groups or group rationality (Owen, 1995, Section X.3-4). This begs the question of whether the coarse core as defined here can also be equivalently recast using the more familiar notion of group rationality.

The problem is that under incomplete information, agents may disagree with respect to their expectations of the value created and value captured by a group of agents. There may even be disagreement within a group itself. Nevertheless, we can define consistency à la group rationality under incomplete information by requiring that every agent in a group believes that the group captures more under a focal distribution than it could create independently (see Appendix B for the formal definition).

It seems intuitive that if a distribution of value is group rational, then blocking groups should not exist. However, our definition of a blocking group under incomplete information requires common certainty that everybody in the blocking group gains. It can be the case that everybody in the blocking group thinks she gains but also thinks that someone else loses. It turns out that this does not occur when: (i) each agent's type mapping is consistent with a common prior; and (ii) no types are ruled out by that prior. Therefore, under those two assumptions, group rationality at every state, together with feasibility, implies the coarse core. We formally state and prove this result in Appendix B. At a conceptual level, Proposition 5 in Appendix B shows that when competitive consistency as usually understood in value capture theory under complete information is suitably extended to incomplete information, then it is consistent with the coarse core.

5.2 Unawareness versus Zero Probability

One may wonder how a model with unawareness differs from a model with a single state space where the events an agent is unaware of are simply assigned zero probability. At a conceptual level, the difference is immediate. Being unaware of a state versus being aware but assigning zero probability to it are two distinct epistemic states of mind. In particular, in the latter case the agent may be willing to bet her entire wealth against a zero probability state, while in the former case, that bet is not even meaningful to him. This is because assigning zero probability to a state means that the agent assigns probability 1 to its non-occurrence.[35]

The idea of betting can be used to behaviorally reveal events of which an agent is unaware and of which he just assigns zero probability. By varying the payoff consequences of a contract in a more expressive space while keeping it constant in less expressive spaces, the modeler can learn from an agent's choices whether she is aware of events in the more expressive space. Schipper (2013, Section 4) suggested this as a method to reveal unawareness and a test for unawareness versus zero probability.

Zero probability and unawareness also have different implications for allocations in the coarse core when we assume that they are private-awareness measurable. Consider the following minimal example: There are two agents, A and B, and two states, ω_1 and ω_2. We consider two analogous situations that are depicted in Figure 11. The left picture shows the information

rationality" elsewhere (e.g., Maschler et al. (2013)).

[35]Stuart Jr. (2017) studies the role of gambles in value creation. This is the only other paper of which we are aware that studies value capture under asymmetric information.

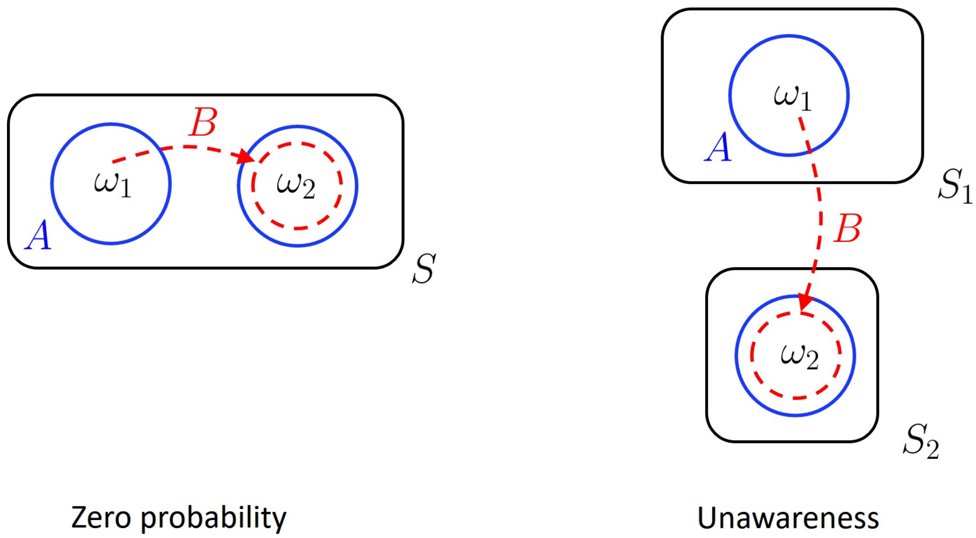

Figure 11: Zero Probability versus Unawareness

structure with zero probability while the right picture shows the unawareness structure. In the zero probability situation to the left, at any state B assigns zero probability to state ω_1. This is indicated by the dashed red arrow emanating from ω_1 and leading to the dashed red information set around ω_2. In the unawareness situation, at any state B is unaware of ω_1, which is indicated by the dashed red arrow emanating from ω_1 in space S_1 and leading to the dashed red information set around ω_2 in space S_2. In both situations, A always knows the state as indicated by his solid-lined blue information sets around each state.

Groups	ω_1	ω_2
{A, B}	100	40
{A}	10	10
{B}	30	30

Table 3: Characteristic Function under Zero Probability and under Unawareness

The characteristic functions are given in Table 3. We print A's maximal allocations in the coarse core in Table 4. First, we observe that A's allocation is affected by the beliefs and awareness of B. Second, the allocations at state ω_1 differ in both situations. While in the case of zero probability, A can achieve a maximum of 100 at ω_1, he can achieve a maximum of only 70 when B is unaware of ω_1. Third, in this example A achieves a higher maximal allocation when agent B assigns zero probability to ω_1 than when B is unaware of ω_1, which at first glance runs counter to intuition.

The reason is that, since B does not perceive ω_1 but only ω_2, A has to guarantee B at least 30 in state ω_1. B would otherwise block the allocation under the belief that he, based on an inability to conceive of a state ω_1 which differs from ω_2, can always earn 30 by working on his own. This is an implication of the private-awareness measurability conditions imposed here on allocations. In contrast, in the zero probability situation, B may explicitly agree to receive zero at state ω_1 because he is certain that ω_1 does not occur and this can be explicitly codified in a contract. An implication is that A may want to raise B's awareness of ω_1 if he expects B to assign a very low probability to ω_1 after becoming aware of it, so as to generate the possibility of profits due to differences in information.[36] Note that in this example, we deliberately used the same characteristic function as in the Apple iTunes case in Section 3.4. This demonstrates that Apple had no incentive to conceal its plans from the music industry. Rather, it profited from the music industry's belief that online music sales would be a low-profit business.

Zero probability	ω_1	ω_2
A	100	10
B	0	30

Unawareness	ω_1	ω_2
A	70	10
B	30	30

Table 4: Value Capture in A's Maximal Profit Distribution in the Coarse Core under Zero Probability and under Unawareness

5.3 Ex-Ante Versus Interim Versus Ex-Post Perspectives

We model agents using the interim perspective, in which the true state specifies each agent's awareness and beliefs prior to the revelation of the state itself. To see that this is the relevant perspective for the extension of cooperative games to incomplete information and unawareness, consider the alternatives. Under the ex-post perspective, the game starts with the state fully resolved. At this point, there is no incomplete information and the approach collapses to cooperative games under complete information.

At the other extreme, consider an ex-ante perspective. Usually ex-ante refers to a situation in which agents have not yet received their private information. Such a situation is often considered under a common prior. That is, all agents hold identical beliefs. This perspective can make no sense of unawareness because each agent's awareness is determined at the interim stage. It would be absurd to assume that everyone is aware of everything ex-ante then, at the interim stage, some agent suffers amnesia and becomes unaware of some events. Thus, consider the ex-ante perspective when every agent has full awareness instead. Then with a common prior all agents have identical ex-ante expectations of all relevant variables. We can now apply the tools of cooperative game theory under complete information except that we use commonly expected values of the characteristic function. Again, this approach essentially boils down to cooperative games under complete information and the traditional approach to value capture use in extant strategy research. If, instead, no common prior is assumed at the ex-ante stage, the situation is no different from our interim setting. Hence, the interim approach is the relevant one when considering extensions of one-shot cooperative games to incomplete information and unawareness.

For business strategy, it will be important to extend our framework to dynamic settings. To do so, it will be necessary to integrate the interim and ex-post perspectives. To see why,

[36] Again, we leave the study of disclosure of awareness in negotiations to a dynamic framework involving bi-form games with unawareness to be presented in future research.

notice that the interim perspective picks up the action at the point at which agents have developed beliefs about their value-creating options but before those beliefs are acted upon and the consequences of those actions are known (the focus of the ex-post perspective). A dynamic framework will require consistency conditions between beliefs and the realities to which they lead. For instance, even if agents agree to work together at an interim stage based upon expectations, it may be necessary to ensure their further participation ex-post, based upon actual outcomes. One idea that has gained traction in strategy and may be promising here is the notion of "subjective equilibrium," which requires consistency between agents' beliefs and reality along the equilibrium path (see Ryall, 2003; Sorenson and Waguespack, 2006; Felin and Foss, 2009; Powell et al., 2011). We leave a systematic study of dynamic unawareness to future work combining ideas from Ryall (2003) and Schipper (2019).

5.4 Implicit Timing and Renegotiation

Although our present framework is static, the characteristic function may implicitly assume some particular timing of events relevant to value creation. This highlights the pros and cons of the characteristic function approach. On the one hand, its detail-freeness requires us to just specify the characteristic function without the need of modeling the minute specifics of non-cooperative interaction. On the other hand, the implicit timing assumed of events affecting value creation need to be taken carefully into account when interpreting characteristic functions.

State	ω_1	ω_2
Common probabilistic belief	0.5	0.5
Willingness to pay for F1's good	150	0
Willingness to pay for F2's good	50	50
Cost of production F1	30	30
Cost of production F2	10	10
Case I: Supplier choice and production before the state resolves		
Value of group $\{F1, F2, B\}$	120	-30
Value of group $\{F1, F1\}$	0	0
Value of group $\{F1, B\}$	120	-30
Value of group $\{F2, B\}$	40	40
Case II: Supplier choice before but production after the state resolves		
Value of group $\{F1, F2, B\}$	120	0
Value of group $\{F1, F1\}$	0	0
Value of group $\{F1, B\}$	120	0
Value of group $\{F2, B\}$	40	40
Case III: Supplier choice and production after the state resolves		
Value of group $\{F1, F2, B\}$	120	40
Value of group $\{F1, F1\}$	0	0
Value of group $\{F1, B\}$	120	0
Value of group $\{F2, B\}$	40	40

Table 5: Implicit Timing Modeled in the Characteristic Function

An example may help to make this more transparent. There are two suppliers, F1 and F2, as well as a buyer B. The buyer wants to buy at most one good from a supplier. Table 5 illustrates how characteristic functions are derived from more basic features like a state-dependent function representing the buyer's willingness to pay for the good of F1 and F2, and costs of production under different assumptions on the implicit timing of contracting and production within the value chain. The good of F1 is either very valuable (at state ω_1) or useless (at state ω_2) to the buyer. In contrast, the good of supplier F2 is always somewhat useful. F1 has higher cost of production than F2. Assume that all groups have common beliefs, assigning equal probability to either state.

Case I The buyer must select the supplier *before* the state is resolved and the supplier's production decision is done *before* the state is resolved. The commonly expected value from the buyer contracting with F1 and F2 being inactive yields 45 while contracting with F2 instead yields 40. Thus, it makes sense for the buyer to contract with F1. Yet, when state ω_2 occurs this yields a loss because production costs are sunk at this time.[37] This illustrates the issue raised in the earlier subsection. In particular, we observe that it might be natural to consider losses in value creation under incomplete information. But those losses could also affect the entire value network when for instance (in a more complex version of the example) the supplier also has contractual obligations to supply to another buyer but at state ω_2 is now unable to do so because of bankruptcy.

Case II The characteristic function features the same willingness to pay and costs of production, but a different timing of production. The buyer still selects the supplier *before* the state is resolved. Yet, now, the firm postpones its decision to produce until *after* the state is resolved (perhaps due to interim market research). Thus, the firm may cancel production when it becomes clear that the buyer is not willing to take the good after all. This saves production costs and eliminates losses.

Case III Retain the willingness to pay and costs of production, but change the timing of contracting *and* production. Assume the buyer can postpone final selection of the supplier until after the state resolved, at which point production also occurs. Even though the buyer may intend to go forward with F1 initially, she may decide to contract with F2 if state ω_2 transpires.

Case III may also arise when considering *renegotiation* in Case II. After it becomes clear in Case II, that the buyer is not interested in F1's good, the market configuration may change when the buyer approaches F2 (in case F2 is still available). So if in Case II commitment to a supplier at the interim stage is "soft" and renegotiation may occur, it is actually more appropriate to consider Case III to begin with.

Clearly, different assumptions about the implicit timing of the market relationships affect modeling the characteristic function. To avoid ambiguities, the modeler needs to consider these issues when specifying the state-contingent characteristic function.

The timing of information revelation, and the importance of irreversibility, to value chain relationships plays an important role in many prior specialized models. For instance, consider a setting with network effects as in Farrell and Saloner (1986). There is one incumbent with an existing network good, and an entrant who may be able to invent a superior network good with probability $\frac{1}{2}$. Two buyers each prefer using the incumbent network to nothing, but prefer the entrant network if it works. If buyers must contract before the entrant network R&D is

[37] We purposefully depart from our framework in order to illustrate a case with losses.

complete, and these contracts are irreversible and cannot be renegotiated, we are in Case I. If renegotiation is possible, or buyers can delay until information about the entrant's network becomes common knowledge to them and the entrant, then we are in Case III.

Likewise, Gill (2008) considers strategic revelation of research results to deter competition who are made to believe their rival is more technologically advanced than they thought. Whether one can deter entry with this information revelation depends critically on implicit assumptions about the timing and renegotiation potential or irreversibility of R&D decisions. The free-form cooperative game approach we pursue in the present paper requires the analyst to consider such details when modelling the characteristic function.

5.5 Information and Awareness Sharing at the Interim Stage

We assume that no information or awareness is shared at the interim stage when agents assess their deals with respect to their alternative opportunities. However, the distributions of value that may arise in the coarse core could give agents reasons to question their beliefs. The coarse core does not require agents to "rationalize" distributions of value. It just requires no agreement by any group to block those distributions. It is a relatively weak solution concept that has its origin in the general equilibrium of exchange economies with asymmetric information (Wilson (1978)), where allocations in the coarse core are best interpreted as proposed by some neutral market maker.

Realistically, though, deals proposed by market participants with private information and awareness may be informative. This has implications for business strategy. Consistent with the resource-based view (see, e.g., Grant, 1991, p. 124), suppose an insider becomes aware that the acquisition of some obscure input has the power to confer it a competitive advantage. The problem is that moving to acquire that input may change transaction patterns and distributions of value in ways that signal the insider status to other market participants, thereby resulting in greater competition for the input. Increased competition erodes the advantage bestowed by insider's superior awareness.

Cooperative game theory does not provide a framework for modeling such "proposed actions" or the resulting signals they provide to other agents. Nevertheless, our framework may already offer a glimpse toward how dynamic updating of information and awareness may play out. For instance, a sequence of unawareness structures could be used to model updates of awareness based on strategic communication among market participants.

We illustrate this with a simple example. Again, let there be two suppliers, F1 and F2, and a buyer B. B wants to buy at most one unit. Initially B is unaware of some events affecting value creation. Both suppliers are fully aware. Suppose there are three awareness levels given by spaces $S_1 \succ S_2 \succ S_3$, all of which are singletons for simplicity (i.e., we are assuming complete information given the awareness level). Initially, B's awareness level is represented by the least expressive space, S_3. This is indicated in the unawareness structure at time $t = 1$ in Figure 12 by the dashed red arrows and information set/support of B's beliefs. The suppliers' information and awareness are given by the blue circles.

At state ω_3, the only state that the buyer considers possible, F1 has an advantage over F2 (see the characteristic function tabulated at the very left of the Figure 12). Thus, B wants to buy from F1. Yet, if B's awareness level were S_2, then he would realize that F2 has an

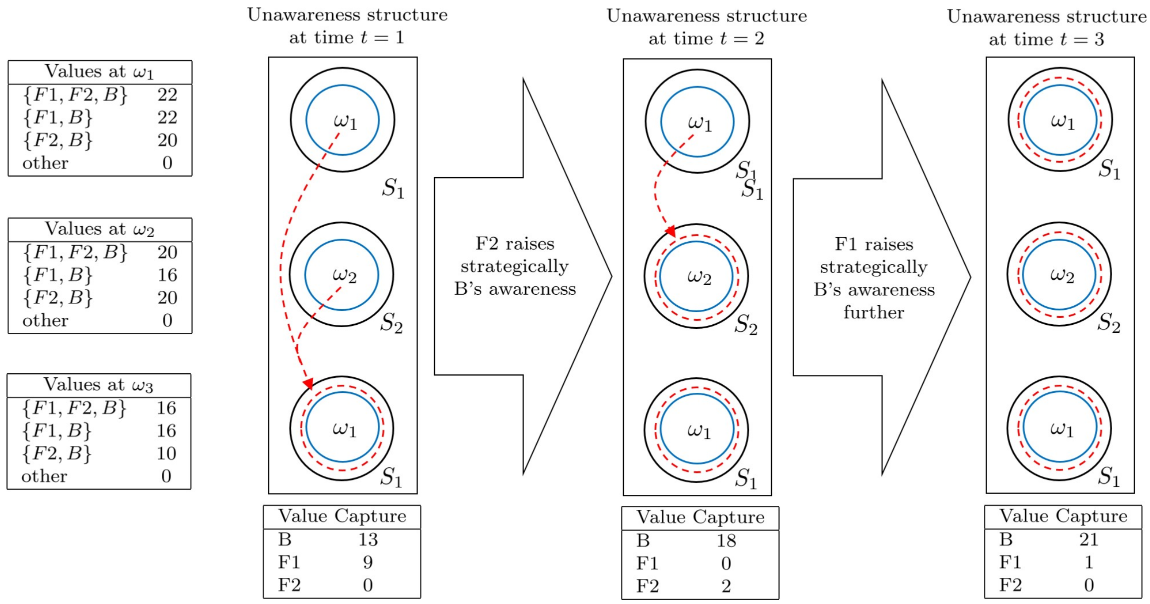

Figure 12: Competition in Awareness

advantage over F1 (see the corresponding characteristic function). It is only natural to assume that F2 would want to raise B's awareness to S_2. Consequently, the "updated" awareness of B is depicted in the unawareness structure at $t = 2$ (at the middle of Figure 12). At this point, if F1 raises B's awareness to S_1, then F1 would have an advantage over F2 once again. (See the unawareness structure at the very right of Figure 12). In this sense, *suppliers compete "in awareness" for the buyer.*

What is the effect of all of this on value capture according to the coarse core? Initially, B is happy with 13. F1, who knows that the value is actually 22, may propose a contract that promises only 13 to B and captures the residual value. Similarly, upon becoming aware of S_2, F2 may offer B 18, who would happily accept that deal, and take the residual for himself (knowing that the actual value created is 20). Finally, at time $t = 3$, B and F1 could agree to split F1's added value equally. *Note that profits go down as suppliers compete in awareness.* Of course, F1 does not have any incentive to preempt F2 by raising B's immediately to S_1. Thus, competition in awareness may drive down awareness rents, but this may also be a gradual process.

A more sophisticated extension of our framework would add a non-cooperative stage before the cooperative stage, in the spirit of bi-form games by Brandenburger and Stuart Jr. (2007). In bi-form games with complete information and unawareness, the persuasive interaction among firms could be modeled explicitly before the cooperative stage takes place. This is left to future research.

5.6 Information Sharing Ex-Post, Incentive Compatibility, Commitment, and Verifiability

We allowed distributions of value in the coarse core to be state-dependent. This may be problematic when the value an agent captures relies upon private information of another agent who may have no incentive, or who may be unable, to reveal this information to the former. In many strategy contexts, incentivizing or otherwise facilitating information revelation may be a central concern. As Alvarez and Barney (2004, p. 622) point out in the context of entrepreneurial strategy, agents who wish to capitalize on rare knowledge must: i) convince the necessary resource providers to partner with them; and ii) prevent the knowledge from diffusing to competitors. On the one hand, Cirque du Soleil may have trouble finding acrobats willing to sign on to a venture based upon what they view as an unworkable concept. On the other hand, if Cirque *can* make them aware of the viability of the concept, what is to prevent these candidate employees from abandoning Cirque in favor of starting their own Cirque-like venture? These concerns also arise in the context of strategic alliances (see e.g., Oxley, 1997; Gulati and Singh, 1998) and, more generally, any setting involving a "marketplace of ideas" (see, e.g., Gans and Stern, 2017).

These considerations led to an active literature on incentive compatible versions of the core for exchange economies with asymmetric information, (see Forges and Serrano (2013) and Forges et al. (2002), for surveys). Essentially, it amounts to molding non-cooperative considerations of incentive compatibility into the cooperative solution concept. This approach does not extend immediately to characteristic function games under incomplete information and unawareness since we take as a primitive the characteristic function rather than individual utility functions with which incentives to reveal information and awareness could be measured. The incentive-compatible core and related solution concepts also require commitment to an incentive compatible mechanism and thus an institution, such as a court of law, that ultimately enforces these commitments. Again, we believe that, rather than overburdening a cooperative solution concept like the coarse core with non-cooperative features, it would be more fruitful to model revelation of information and awareness explicitly in a prior non-cooperative stage. This seems most straightforward to do via an extension of our work to bi-form games with incomplete information and unawareness.

For now, concerns with incentive compatibility can be alleviated by assuming that states are verifiable ex-post. While this is not without loss of generality, we argue that there are many situations of interest in which states do eventually become public. For instance, it did eventually become obvious that Cirque's reinvention of the circus was a hit. Assuming that the state becomes verifiable ex-post does beg the question "In which state space?" when unawareness is present. The natural answer is: the space that results when awareness of all agents is pooled. This is not necessarily the state in the most refined space (which would imply that all agents become aware of everything ex-post).

5.7 Awareness of Unawareness

In many situations, agents have some sense that there is "something" out there of which they are unaware. We may refer to this as "awareness of unawareness". Our notion of (un)awareness is propositionally determined in the sense that awareness is about basic events like "penicillin has antibiotic properties". In unawareness structures, if an agent is unaware of an event, then she is unaware that she is unaware of the event. It does not rule out a situation where the agent has some sense that there is "something" of which she is unaware. While there have been models of awareness of unawareness in epistemic logic (see Schipper (2015) for a survey), all those approaches are not syntax-free and, hence, are difficult to adapt directly to game theoretic settings.

Our approach is essentially neutral to a general awareness of unawareness. Agents take everything of which they are aware into account. They may have a vague sense of unawareness. Yet, in order for this sense of unawareness to have any effect, agents should either have some actions available to investigate their unawareness (e.g., asking a lawyer, a doctor, or some other kind of expert who could be aware of it), or they should have an attitude towards these feelings that affects how they rank their available actions (since different actions may expose them to different unawareness levels). In any case, in order to tackle this type of analysis, a non-cooperative stage in which agents can take actions to address their vague sense of unawareness is required. Yet again, this motivates an extension of our approach to bi-form games with unawareness, where a non-cooperative stage would use the formalism of extensive-form games with unawareness developed by Heifetz et al. (2013b). The latter allows for awareness of unawareness as well as surprises and changes of awareness.

5.8 Closing Thoughts

Where do we stand? Private information, awareness of opportunities, entrepreneurial foresight: all are surely of first-order importance in explaining the differential profitability of firms. These concepts have not been easy to incorporate into existing formal theories of firm performance. We show how to extend one particularly well-known model of firm performance - the value capture model - to incorporate information and awareness differences. Our extension uses tools developed in logic, decision theory, and game theory.

Nonetheless, the model here is only a first step. While we have developed a theory of value capture under known and unknown unknowns in the static case, the preceding discussion makes clear that extending the model to dynamic settings appears to be the most promising direction. Such an extension could address implicit timing issues, the exploration of strategic revelation of information and awareness, and awareness of unawareness.

We suggest that the most promising path for incorporating these issues will require explicitly modeling strategic information and awareness transmission. For the reasons described above, a model based solely on cooperative game solutions with characteristic functions is limited in its ability to incorporate important aspects of that communication. We therefore argue that theorists should focus on extending the bi-form game approach of Brandenburger and Stuart Jr. (2007) by introducing a non-cooperative dynamic stage in the spirit of Heifetz et al. (2013b) before the cooperative stage. This is left for future research.

A Formal Details

In this section we provide a brief but rigorous and self-contained exposition of our general static theory of value capture under known and unknown unknowns. This, of course, contains also the static theory of value capture under known unknowns as a special case.

Denote by $\langle \mathcal{S}, \succeq \rangle$ the nonempty finite lattice of nonempty finite disjoint state-spaces and let $\Omega := \bigcup_{S \in \mathcal{S}} S$ the union of all state spaces. For any $S, S' \in \mathcal{S}$ with $S' \succeq S$, there is a surjective projection $r_S^{S'} : S' \longrightarrow S$ for which r_S^S is the identity for any $S \in \mathcal{S}$. Moreover, projections commute, i.e., for any $S, S', S'' \in \mathcal{S}$ with $S'' \succeq S' \succeq S$, we have $r_S^{S''} = r_S^{S'} \circ r_{S'}^{S''}$. For any $\omega \in S$ and $S' \succeq S$, the inverse image $(r_S^{S'})^{-1}(\omega)$ denotes the *ramifications in* S' of state ω.

For a subset of states $D \subseteq S$, for some space $S \in \mathcal{S}$, denote by $D^{\uparrow} := \bigcup_{S' \succeq S} \left(r_S^{S'} \right)^{-1}(D)$. An *event* has now the form $E = D^{\uparrow}$ with $D \subseteq S$, for some $S \in \mathcal{S}$. D is called the *base* of the event E and S the *base-space* of the event E denoted by $S(E)$. If $E \neq \emptyset$, then S is uniquely determined by E. Otherwise, we write $\emptyset^S$ for the vacuous event that is based in space S. To understand this, note that the empty set is a subset of any state space. When we take the empty subset of a state space, we can consider also the union of its inverse images in more expressive spaces, which of course is empty as well. This is a vacuous event. But all these vacuous events are different because they have different base spaces. While this may look strange at first, it makes perfect sense. A vacuous event corresponds to a contradiction, a description that is contradictory like "the sun is shining and the sun is not shining". There is no state of the world where this is true. However, contradictions can be more or less rich depending on how rich is the language with which they are described. This is essentially specified with the base space, and that's why we must have different vacuous events. We denote by Σ the set of events. We mention that although Σ is not an algebra (because it can have more than one vacuous event) it essentially "works" like an algebra, which comes in handy when we define beliefs.

For any state space $S \in \mathcal{S}$, let $\Delta(S)$ be the set of probability measures S. We consider this set itself as a measurable space endowed with the σ-field $\mathcal{F}_{\Delta(S)}$ generated by the sets $\{\mu \in \Delta(S) : \mu(D) \geq p\}$, where $D \in 2^S$ and $p \in [0, 1]$. In order model beliefs at different levels of awareness, we need to relate probability measures on a richer space to probability measures on poorer spaces. Formally, for a probability measure $\mu \in \Delta(S')$, the marginal $\mu_{|S}$ of μ on $S \preceq S'$ is defined by

$$\mu_{|S}(D) := \mu\left(\left(r_S^{S'} \right)^{-1}(D) \right), \quad D \in 2^S.$$

To extend probability measures to events of our lattice structure, let S_μ denote the space on which μ is a probability measure. Whenever for some event $E \in \Sigma$ we have $S_\mu \succeq S(E)$ (i.e., the event E can be expressed in space S_μ) then we abuse notation slightly and write

$$\mu(E) = \mu(E \cap S_\mu).$$

If $S(E) \npreceq S_\mu$ (i.e, the event E is not expressible in the space S_μ because either S_μ is strictly poorer than $S(E)$ or S_μ and $S(E)$ are incomparable), then we say that $\mu(E)$ is undefined.

Since we want to allow also for incomplete information and unawareness of the existence of players, we introduce the "existence" correspondence $\mathcal{E} : N \longrightarrow \Sigma$ that assigns to each agent $i \in N := \{1, ..., n\}$ an event in which she exists. Moreover, let $\mathcal{S}_i := \{S \in \mathcal{S} : \mathcal{E}(i) \cap S \neq \emptyset\}$ be the complete sublattice of spaces with states in which player i exists.

For each agent $i \in N$ there is a *type mapping* $t_i : \mathcal{E}(i) \longrightarrow \bigcup_{S \in \mathcal{S}_i} \Delta(S)$. We impose the following properties:

(i) *Confinement:* If $\omega \in S' \cap \mathcal{E}(i)$ then $t_i(\omega) \in \triangle(S)$ for some $S \preceq S'$. This property has already been discussed in the main text.

(ii) If $S'' \succeq S' \succeq S$, $S'', S', S \in \mathcal{S}_i$, $\omega \in S'' \cap \mathcal{E}(i)$, and $t_i(\omega) \in \triangle(S')$ then $t_i(\omega_S) = t_i(\omega)_{|S}$. Property (ii) compares the types of an agent in a state $\omega \in S'$ and its projection to ω_S, for some less expressive space $S \preceq S'$. Suppose an agent's awareness level at ω is S', which means that agent i's belief at state ω is over states in S'. What should the agent's beliefs be at a poorer description of ω at an awareness level S below S'? Property (ii) says that the agent should hold the same belief over an event E as he does at ω provided that he is still aware of the event E. In this sense, the types at ω and ω_S just differ in their awareness.

(iii) If $S'' \succeq S' \succeq S$, $S'', S', S \in \mathcal{S}_i$,, $\omega \in S'' \cap \mathcal{E}(i)$, and $t_i(\omega_{S'}) \in \triangle(S)$ then $S_{t_i(\omega)} \succeq S$. Property (iii) also compares the types of an agent in a state $\omega \in S'$ and its projection to ω_S, for some less expressive space $S \preceq S'$. Property (iii) means that at ω an agent cannot be unaware of an event that she is aware of at the projected state $\omega_{S'}$.

(iv) Introspection: For every $\omega \in \mathcal{E}(i)$, $t_i(\omega)\left(\left\{\omega' \in \mathcal{E}(i) : t_i(\omega')_{|S_{t_i(\omega)}} = t_i(\omega)\right\}\right) = 1$. Property (iv) means that at every state, agent i is certain about her own beliefs. More precisely, for every state ω, the type of agent i at ω is certain of the set of states at which agent i's type or the marginal thereof coincides with her type at ω. This property implies what is called introspection (i.e., Property (va) in Proposition 4 in Heifetz et al. (2013a)). It rules out mistakes in information processing.

Note that an agent has only beliefs at states in which the agent exists.

We denote by $\left\langle \langle \mathcal{S}, \succeq \rangle, \left(r^{S_\alpha}_{S_\beta}\right)_{S_\beta \preceq S_\alpha}, \mathcal{E}, (t_i)_{i \in N} \right\rangle$ a finite interactive unawareness structure. This completes the model of beliefs and awareness of groups.

Next, we define characteristic function games with incomplete information and unawareness. For every state $\omega \in \Omega$, let $N(\omega) := \{i \in N : \omega \in \mathcal{E}(i)\}$. $N(\omega)$ is the set of agents that exist at state $\omega \in \Omega$. Further, we let $\mathcal{G}(\omega) := \{N' \subseteq N : N' \subseteq N(\omega), N' \neq \emptyset\}$ be the set of all nonempty subsets of agents that exist at ω.

The characteristic function $v : \bigcup_{\omega \in \Omega} \{\omega\} \times \mathcal{G}(\omega) \longrightarrow \mathbb{R}_+$ assigns to each state $\omega \in \Omega$ and each subset of agents $G \in \mathcal{G}(\omega)$ who exist at state ω a value $v(\omega, G)$ which is some non-negative real number.

Definition 7 *A finite cooperative game with incomplete information and unawareness* $\left\langle N, v, \langle \mathcal{S}, \succeq \rangle, \left(r^S_{S'}\right)_{S' \preceq S}, \mathcal{E}, t_1, ..., t_n \right\rangle$ *consists of*

- *a finite set of agents* $N := \{1, ..., \}$,
- *a finite unawareness structure* $\left\langle \mathcal{S}, \left(r^S_{S'}\right)_{S' \preceq S}, \mathcal{E}, t_1, ..., t_n \right\rangle$, *where* $\mathcal{S}$ *lattice of finite state spaces with projections* $r^{S'}_S : S' \longrightarrow S$ *for any* $S' \succeq S$, $S, S' \in \mathcal{S}$, $\Omega = \bigcup_{S \in \mathcal{S}} S$, $\mathcal{E} : N \longrightarrow \Sigma$ *is the existence correspondence, and for each agent* $i \in N$, $t_i : \mathcal{E}(i) \longrightarrow \bigcup_{S \in \mathcal{S}_i} \Delta(S)$ *is a type mapping satisfying properties (i) to (iv).*
- *a state-contingent characteristic function* $v : \bigcup_{\omega \in \Omega} \{\omega\} \times \mathcal{G}(\omega) \longrightarrow \mathbb{R}_+$ *with* $\mathcal{G}(\omega) = \{N' \subseteq N : N' \subseteq N(\omega), N' \neq \emptyset\}$ *and* $N(\omega) := \{i \in N : \omega \in \mathcal{E}(i)\}$.

A state-dependent distribution of payoffs is $\pi(\omega) = (\pi_i(\omega))_{i \in N(\omega)}$ with $\pi_i : \mathcal{E}(i) \longrightarrow \mathbb{R}_+$ for all $i \in N$. $\pi_i(\omega)$ represents agent i's payoff in state $\omega \in \mathcal{E}(i)$. Agents receive payoffs only at states in which they exist.

Fix a cooperative game with incomplete information and unawareness, $\left\langle N, v, \langle \mathcal{S}, \succeq \rangle, \left(r^{S_\alpha}_{S_\beta}\right)_{S_\beta \preceq S_\alpha}, \mathcal{E}, (t_i)_{i \in N} \right\rangle$.

Definition 8 (Feasibility) *We say that a distributions of payoffs π is* feasible for group G *if*

$$\sum_{i \in G} \pi_i(\omega) \leq v(\omega, G) \text{ for all } \omega \in \Omega \text{ for which } G \in \mathcal{G}(\omega).$$

We say that π is feasible *if it is feasible for all groups $G \in \mathcal{G} := \{N' \subseteq N : N' \neq \emptyset\}$. Further, we say that π is* feasible for all existing agents *if for all $\omega \in \Omega$,*

$$\sum_{i \in N(\omega)} \pi_i(\omega) \leq v(\omega, N(\omega)).$$

The payments to agents of group G at state ω cannot exceed the value created by the group at state ω for all states at which this group exists.

Next we want to define blocking groups. To this extend, we need to formalize what is common belief among a group. For each $i \in N$, define the certainty operator $B^1_i : \Sigma \longrightarrow \Sigma$ on events by for all $E \in \Sigma$

$$B^1_i(E) = \{\omega \in \Omega : t_i(\omega)(E) = 1\}$$

if there is a state such that $t_i(\omega)(E) = 1$, and by $B^1_i(E) = \emptyset^{S(E)}$ otherwise. For each event $E \in \Sigma$, $B^1_i(E)$ is the set of states in which agent i is certain (i.e., assigns probability 1) to the event E. By Heifetz et al. (2013a, Prop. 2), for any event $E \in \Sigma$, $B^1_i(E)$ is an $S(E)$-based event in Σ. That is, the certainty operation is well-behaved and easy to work with. Note that by definition $B^1_i(E) \subseteq \mathcal{E}(i)$ since beliefs of agent i are only defined for states at which she exists.

For any group $G \in \mathcal{G}$ define the mutual certainty operator by $B^1_G(E) = \bigcap_{i \in G} B^1_i(E)$. That is, in every state of $B^1_G(E)$ every agent in group G is certain of the event E. Again, the mutual certainty operator is tractable because for every event $E \in \Sigma$ and group G, the set of states $B^1_G(E)$ is an $S(E)$-based event in Σ. Moreover, by definition for any event $E \in \Sigma$, $B^1_G(E) \subseteq \cap_{i \in G} \mathcal{E}(i)$.

We can now iterate the mutual certainty operator to formalize that not just everybody in G is certain of the event E but also everybody in G is certain of that fact, and everybody in G is certain of this as well etc. For any group $G \in \mathcal{G}$ define the common certainty operator by $CB^1_G(E) = \bigcap_n^\infty (B^1_G)^n(E)$. Again, for any event $E \in \Sigma$ and group G, the set $CB^1_G(E)$ is an $S(E)$-based event and $CB^1_G(E) \subseteq \bigcap_{i \in G} \mathcal{E}(i)$. For properties of these operators, see Heifetz et al. (2013a, Propositions 4-7) for unawareness-belief structures and see Monderer and Samet (1989) for standard type spaces.

Remark 1 *For any event $E \in \Sigma$ and group of agents $G \in \mathcal{G}$,*

$$CB^1_G(E) \subseteq CB^1_G\left(\bigcap_{i \in G} \mathcal{E}(i)\right).$$

Proof. For any event $E \in \Sigma$ and group of agents $G \in \mathcal{G}$, $CB^1_G(E) = CB^1_G(CB^1_G(E))$ by introspection. By definition of type mappings, $CB^1_G(E) \subseteq \bigcap_{i \in G} \mathcal{E}(i)$. By monotonicity of beliefs (Heifetz et al., 2013b, Prop. 4), $CB_G^1(E) \subseteq \bigcap_{i \in G} \mathcal{E}(i)$ implies $CB_G^1(CB_G^1(E)) \subseteq CB_G^1\left(\bigcap_{i \in G} \mathcal{E}(i)\right)$. Now the conclusion follows. □

That is, common certainty of an event among G implies common certainty of the existence of group G.

Given a state-dependent distribution of payoffs π and a state $\omega \in \Omega$, any agent $i \in N(\omega)$ can form expectations about the sum of payoffs that a group $G \in \mathcal{G}$ receives. We denote by

$$\mathbb{E}\left[\pi_i \mid t_i(\omega)\right] := \sum_{\omega' \in \Omega} \pi_i(\omega') \cdot t_i(\omega)(\{\omega'\})$$

agent i's (conditional/interim) expected payment conditional on state $\omega \in \mathcal{E}(i)$. By Confinement we could write the right-hand side by summing over $\omega' \in S_{t_i(\omega)}$ instead the entire Ω.

For any group $G \in \mathcal{G}$, define the set of states[38]

$$[\pi' \ G\text{-dominates } \pi] := \bigcap_{i \in G} \left\{\omega \in \Omega : \mathbb{E}[\pi_i' \mid t_i(\omega)] > \mathbb{E}[\pi_i \mid t_i(\omega)]\right\}$$

By the definition of type mapping, we have $[\pi' \ G\text{-dominates } \pi] \subseteq \bigcap_{i \in G} \mathcal{E}(i)$.

Definition 9 (Blocking Group) *For a profile of payoff functions π that is feasible, a group G blocks π at state $\omega \in \Omega$ if there exists a profile of payoff functions π' that is feasible for group G such that $\omega \in CB_G^1([\pi' \ G\text{-dominates } \pi])$.*

A profile of payoff functions π that is feasible for the entire set of agents N is blocked by group G at state $\omega \in \Omega$ if there exists a profile of payoff functions π' that is feasible for group G and at ω it is common certainty among group G that π' yields each member of G a strictly higher expected payment than π. Note that by definition, if group G blocks π at state $\omega \in \Omega$, then $G \in \mathcal{G}(\omega)$. That is, the group G must exist at state ω. It follows from Remark 1 above that if agents $i, j \in G$ but at ω agent i does not believe that j exists, then G cannot block any payoff distribution at ω.

Definition 10 (Coarse Core) *The profile of payoff functions π is in the coarse core if and only if*

(i) π is feasible for all existing agents, and

(ii) for all $\omega \in \Omega$ no group blocks π.

It follows trivially from the definitions that if a group does not exist at ω, it cannot block any payoff distribution at state ω. So effectively, at every state $\omega \in \Omega$ only groups of existing agents need to be taken into account. Note that feasibility applies to all existing agents. That is, at any state the distribution of payoffs among all agents that exist at that state cannot exceed the value created by all those agents at that state.

[38]Note that these sets of states may not be events in the unawareness belief structure. Yet, the belief operators can be extended in a straightforward way to those sets. See Heifetz et al. (2013a).

B Group Rationality

In this section, we investigate the relationship between group rationality and no blocking group as discussed in Section 5.1. For this, we assume for simplicity that all agents exist in every state. I.e., for all $i \in N$, $\mathcal{E}(i) = \Omega$. This eliminates unawareness of the existence of agents.

Definition 11 (Group Rationality) *A profile of payoff functions π is G-group rational at $\omega \in \Omega$ if for all $i \in G$,*

$$\mathbb{E}\left[\sum_{j \in G} \pi_j \mid t_i(\omega)\right] \geq \mathbb{E}\left[v(G, \cdot) \mid t_i(\omega)\right].$$

A profile of feasible payoff functions π is group rational at $\omega \in \Omega$ if it is G-group rational for every group $G \in \mathcal{G}$.

Distribution π is G-group rational at ω if every agent in group G expects at ω that the group earns at least its value.

It seems intuitive that if a profile of payoff functions is group rational then there should not be a blocking group. Yet, for a blocking group there is common certainty that everybody in the blocking group gains. But it can be the case that everybody in the blocking group thinks she gains but also thinks that someone else loses. These cases are ruled out when the agents' beliefs are somehow consistent (even though agents may have different awareness) as implied by the common prior assumption.

To define a common prior on unawareness-belief structures we require some notation. Typically, a prior is interpreted as a probability measure from which the agents' beliefs are derived. Yet, agents may be unaware of some events. That's why we need a more explicit formalism that allows us to model what events agents are aware of. For each agent $i \in N$, define the awareness operator on events $E \in \Sigma$ by

$$A_i(E) := \{\omega \in \Omega : t_i(\omega) \in \Delta(S), S \succeq S(E)\}$$

if there is a state $\omega \in \Omega$ such that $S_{t_i(\omega)} \succeq S(E)$, and by $\emptyset^{S(E)}$ otherwise. That is, agent i is aware of event E in all states in which her belief is defined on a space that can express the event E. Heifetz et al. (2013a, Proposition 1) show that for any agent $i \in N$ and event $E \in \Sigma$, $A_i(E)$ is an $S(E)$-based event in Σ. Moreover, Heifetz et al. (2013a, Proposition 5 and 7) show that indeed it models non-trivial awareness in unawareness structures and captures properties of awareness introduced in the prior literature.

In a standard state space, a prior is a probability measure on the space. In unawareness-beliefs structures there is a collection of spaces with an order structure. That why in our context a *prior* for agent i is a system of probability measures $P_i = \left(P_i^S\right)_{S \in \mathcal{S}} \in \prod_{S \in \mathcal{S}} \Delta(S)$ such that

1. The system is projective: If $S' \preceq S$ then the marginal of P_i^S on S' is $P_i^{S'}$. (That is, if $E \in \Sigma$ is an event whose base-space $S(E)$ is lower or equal to S', then $P_i^S(E) = P_i^{S'}(E)$.)
2. Each probability measure P_i^S is a convex combination of i's beliefs in S: For every event $E \in \Sigma$ such that $S(E) \preceq S$,

$$P_i^S\left(E \cap S \cap A_i(E)\right) = \sum_{\omega \in S \cap A_i(E)} t_i(\omega)(E)\, P_i^S(\{\omega\}).$$

This conditions is the analogue to the typical assumption that an agent's belief is the prior conditioned on her information.

$P = \left(P^S\right)_{S \in \mathcal{S}} \in \prod_{S \in \mathcal{S}} \Delta(S)$ is a *common prior* among group $G \in \mathcal{G}$ if P is a prior for every agent $j \in G$.

In principle, the prior may assign zero probability to some types of agents. It is convenient to rule this out. Denote for any $i \in N$ and $\omega \in \Omega$, $[t_i(\omega)] := \{\omega' \in \Omega : t_i(\omega') = t_i(\omega)\}$. This is the set of states in which agent i has the type $t_i(\omega)$. A common prior $P = \left(P^S\right)_{S \in \mathcal{S}} \in \prod_{S \in \mathcal{S}} \Delta(S)$ is *positive* if and only if for all $i \in N$ and $\omega \in \Omega$: If $t_i(\omega) \in \triangle(S')$, then $P^S\left(\left([t_i(\omega)] \cap S'\right)^{\uparrow} \cap S\right) > 0$ for all $S \succeq S'$.

The relation between group rationality and the coarse core can now be stated. The proof applies the No-speculative-betting theorem under unawareness (Heifetz et al. (2013a, Theorem 1)).

Proposition 5 *Assume a positive common prior. If a profile of feasible payoff functions π is group rational at every state $\omega \in \Omega$, then π is in the coarse core.*

Proof. If a profile of payoff functions π is feasible for every group $G \in \mathcal{G}$ and group rational at every state $\omega \in \Omega$, then for all $\omega \in \Omega$ and all groups $G \in \mathcal{G}$,

$$\mathbb{E}\left[\sum_{j \in G} \pi_j \mid t_i(\omega)\right] \geq \mathbb{E}\left[v(G, \cdot) \mid t_i(\omega)\right] \text{ for all } i \in G.$$

By Jensen's Inequality, for all $\omega \in \Omega$ and groups $G \in \mathcal{G}$,

$$\sum_{j \in G} \mathbb{E}\left[\pi_j \mid t_i(\omega)\right] \geq \mathbb{E}\left[v(G, \cdot) \mid t_i(\omega)\right] \text{ for all } i \in G. \tag{1}$$

Suppose by contradiction that π is not in the coarse core. Then there exists $\omega \in \Omega$, a group $G \in \mathcal{G}$, and a profile of payoff functions π' feasible for group G such that $\omega \in CB^1_G([\pi' \ G - \text{dominates } \pi])$. $\omega' \in [\pi' \ G - \text{dominates } \pi]$ if and only if

$$\mathbb{E}[\pi'_i \mid t_i(\omega')] > \mathbb{E}[\pi_i \mid t_i(\omega')] \text{ for all } i \in G.$$

Suppose that there exists $\omega \in CB^1_G([\pi' \ G - \text{dominates } \pi])$ and $i \in G$ such that

$$\mathbb{E}[\pi'_j \mid t_i(\omega)] \leq \mathbb{E}[\pi_j \mid t_i(\omega)] \text{ for some } j \neq i \text{ with } j \in G.$$

Then

$$\omega \in CB^1_{\{i,j\}}\left(\{\omega \in \Omega : \mathbb{E}[\pi'_j \mid t_i(\omega)] \leq \mathbb{E}[\pi_j \mid t_i(\omega)]\} \cap \{\omega \in \Omega : \mathbb{E}[\pi'_j \mid t_j(\omega)] > \mathbb{E}[\pi_j \mid t_j(\omega)]\}\right).$$

Since the unawareness-beliefs structure satisfies a positive common prior, this contradicts the "No-speculative-betting" theorem of Heifetz et al. (2013a, Theorem 1). Hence, we must have that for all $\omega \in CB^1_G([\pi' \ G - \text{dominates } \pi])$ and $i \in G$

$$\mathbb{E}[\pi'_j \mid t_i(\omega)] > \mathbb{E}[\pi_j \mid t_i(\omega)] \text{ for all } j \in G.$$

Summing for all $j \in G$, we obtain for all $\omega \in CB^1_G([\pi' \; G-\text{dominates } \pi])$ and $i \in G$

$$\sum_{j \in G} \mathbb{E}[\pi'_j \mid t_i(\omega)] > \sum_{j \in G} \mathbb{E}[\pi_j \mid t_i(\omega)]. \tag{2}$$

Since π' is feasible for group G, we have for all $\omega \in \Omega$,

$$\sum_{i \in G} \pi'_i(\omega) \leq v(G, \omega).$$

This implies (by linearity of the expectations operator)

$$\sum_{j \in G} \mathbb{E}[\pi'_j \mid t_i(\omega)] \leq \mathbb{E}[v(G, \cdot) \mid t_i(\omega)] \text{ for all } i \in N, \omega \in \Omega.$$

Together with Inequality (1) it implies for all $\omega \in \Omega$,

$$\sum_{j \in G} \mathbb{E}\left[\pi_j \mid t_i(\omega)\right] \geq \sum_{j \in G} \mathbb{E}\left[\pi'_j \mid t_i(\omega)\right] \text{ for all } i \in G. \tag{3}$$

This yields a contradiction to Inequality (2). □

C Proofs

In this section, we state formally and prove the propositions in the main text.

Proposition 2 (Formal restatement) *Suppose π is in the coarse core. Then for any $i \in N$,*

$$CB^1\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}\right) = \emptyset.$$

Proof. Suppose by contradiction that there exist $i \in N$ such that

$$CB^1\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}\right) \neq \emptyset.$$

It follows that

$$CB^1_{N \setminus \{i\}}\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}\right) \neq \emptyset.$$

Now,

$$CB^1_{N \setminus \{i\}}\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}\right) =$$

$$CB^1_{N\setminus\{i\}}\left(\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}[v(N\setminus\{i\})\mid t_j(\omega)]>\mathbb{E}[v(N)-\pi_i\mid t_j(\omega)]\}\right)$$

Since π is in the coarse core, π is feasible. Thus

$$CB^1_{N\setminus\{i\}}\left(\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}[v(N\setminus\{i\})\mid t_j(\omega)]>\mathbb{E}[v(N)-\pi_i\mid t_j(\omega)]\}\right)=$$
$$CB^1_{N\setminus\{i\}}\left(\bigcap_{j\in N}\left\{\omega\in\Omega:\mathbb{E}[v(N\setminus\{i\})\mid t_j(\omega)]>\mathbb{E}\left[\sum_{k\in N\setminus\{i\}}\pi_k\mid t_j(\omega)\right]\right\}\right).$$

Given π, construct an alternative distribution π' by for all $j\in N\setminus\{i\}$ and $\omega\in\Omega$,

$$\pi'_j(\omega)=\pi_j(\omega)+\frac{v(N\setminus\{i\},\omega)-\sum_{k\in N\setminus\{i\}}\pi_k(\omega)}{n-1}.$$

Clearly, π' is feasible for $N\setminus\{i\}$. I.e., summing over agents in $N\setminus\{i\}$ we have for all $\omega\in\Omega$,

$$\begin{aligned}\sum_{j\in N\setminus\{i\}}\pi'_j(\omega) &= \sum_{j\in N\setminus\{i\}}\pi_j(\omega)+(n-1)\frac{v(N\setminus\{i\},\omega)-\sum_{k\in N\setminus\{i\}}\pi_k(\omega)}{n-1}\\ &= v(N\setminus\{i\},\omega)\end{aligned}$$

Now

$$\begin{aligned}&CB^1_{N\setminus\{i\}}\left(\bigcap_{j\in N}\left\{\omega\in\Omega:\mathbb{E}[v(N\setminus\{i\})\mid t_j(\omega)]>\mathbb{E}\left[\sum_{k\in N\setminus\{i\}}\pi_k\mid t_j(\omega)\right]\right\}\right)=\\ &CB^1_{N\setminus\{i\}}\left(\bigcap_{j\in N}\left\{\omega\in\Omega:\mathbb{E}\left[\frac{v(N\setminus\{i\})-\sum_{k\in N\setminus\{i\}}\pi_k}{n-1}\mid t_j(\omega)\right]>0\right\}\right)\\ \subseteq\ &CB^1_{N\setminus\{i\}}\left(\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}\left[\pi'_j(\omega)>\pi_j(\omega)\mid t_j(\omega)\right]>0\}\right)\end{aligned}$$

But last line means that $N\setminus\{i\}$ blocks π, a contradiction to π being in the coarse core. □

Proposition 4 (Formal restatement) *Suppose π is in the coarse core. Then for any $i\in N$,*

$$CB^1\left(\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}[\pi_i\mid t_j(\omega)]>0\}\cap\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}[v(N)-v(N\setminus\{i\})\mid t_j(\omega)]=0\}\right)=\emptyset.$$

Proof. Note that

$$\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}[\pi_i\mid t_j(\omega)]>0\}\cap\bigcap_{j\in N}\{\omega\in\Omega:\mathbb{E}[v(N)-v(N\setminus\{i\})\mid t_j(\omega)]=0\}\subseteq$$

$$\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}$$

Since the common belief operator satisfies monotonicity, i.e., $E \subseteq F$ implies $CB^1(E) \subseteq CB^1(F)$ (follows from monotonicity of belief, see Heifetz et al. (2013a, Prop. 4)), we have

$$\begin{aligned} & CB^1\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > 0\} \cap \bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)] = 0\}\right) \\ \subseteq \quad & CB^1\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}\right) \end{aligned}$$

By Proposition 2, we must have for π in the coarse core

$$CB^1\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)]\}\right) = \emptyset.$$

Hence,

$$CB^1\left(\bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[\pi_i \mid t_j(\omega)] > 0\} \cap \bigcap_{j \in N} \{\omega \in \Omega : \mathbb{E}[v(N) - v(N \setminus \{i\}) \mid t_j(\omega)] = 0\}\right) = \emptyset.$$

□

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